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Teacher Beliefs and the Common Core State Standards for Mathematics

A DISSERTATION
SUBMITTED TO THE FACULTY OF THE GRADUATE SCHOOL
OF HAMLINE UNIVERSITY
BY

Erin M. Smith

IN PARTIAL FULFILLMENT OF THE REQUIREMENTS
FOR THE DEGREE OF
DOCTOR OF EDUCATION

Dr. Charlayne Myers

July, 2015

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Dedication

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CHAPTER ONE: INTRODUCTION

“Mathematics education is much more complicated than you expected, even though you expected it to be more complicated than you expected.”

(Edward Begle, 1968)

Introduction

The notion of mathematics standards is as old a concept as mathematics education itself. For millennia, mathematics has been part of the natural and cultural fabric of the human experience. Although no one knows exactly when and how mathematics began, every civilization that developed writing also left behind evidence of mathematical knowledge in the form of artifacts. These artifacts, the most ancient found in Africa dating back 37,000 years, reveal knowledge of number, form, and operations that seem to be “part of the common heritage of humanity everywhere” (Berlinghoff & Gouvêa, 2004, p. 6). Equally important, these artifacts reveal the desire to teach mathematical knowledge to the next generation, for the purposes of practical application and the pure pleasure of intellectual discovery.

As civilizations formally and informally decide what knowledge will be passed on, they establish standards—model levels of acceptable quality or achievement—by authority, custom, or consent. Education standards are therefore norms or culturally agreed upon ideas of what is to be learned (and sometimes how it is to be learned) which reflect the human desire to progress individually and collectively by creating a *core* of knowledge *common* among learned members of society.

These two words—*core* and *common*—have sparked a controversy in American education that is at once both modern and antiquated. This research was designed to answer two questions about teachers’ mathematical beliefs and practices related to the recent Common Core Standards for Mathematics (CCSSM) (NGA & CCSSO, 2010):

- 1) *What do teachers believe about mathematics, teaching mathematics, and learning mathematics?*
- 2) *How do these teacher beliefs influence their interpretation and implementation of the Common Core State Standards for Mathematics?*

Determining what is considered important for the next generation to know, and how best to teach and learn it, is a complex endeavor; those with the power to choose and the decisions they make evolve across space and time.

As mathematics began to play an increasingly important role in society—from early architecture and navigation, through mercantile economies and the industrial revolution, to current applications in diverse fields such as computers, biostatistics, and astrophysics—the need for all members of society to have a solid foundation in mathematics has increased as well. Not only can mathematical prowess contribute to economic and military security on a national level, a sound mathematics education can also be a democratizing force on an individual level. Civil rights leader turned teacher Bob Moses (2012) asserts “the information technologies have...shifted the quantitative literacy needed for citizenship from arithmetic to algebra” (Letter, para. 19). How have American students performed in this important discipline?

The launching of Sputnik and poor performance of American mathematics students on the international stage (McKnight et al., 1987) placed mathematics education on the national agenda more than half a century ago. While assessment data, such as the Trends in International Mathematics and Science Study (TIMSS), show improvement in the average mathematics scores achieved at both the fourth and eighth grade levels in the period from 1995 to 2011 (IES, 2014), the data also reflects the disparity of mathematics performance between American students and their international peers. Closer examination of assessment data reveals an alarming achievement gap between certain classes of American students (Darling-Hammond, 2010). Although the National Assessment of Educational Progress (NAEP) indicates improvement by nearly all student groups in mathematics (Ravitch, 2013), the disappointing performance of U.S. students internationally and the mathematical achievement gap within specific groups of American students has led to a variety of mathematics education reforms.

Despite consensus, within and outside the education community, that mathematics education in the United States warrants improvement, there has not been agreement on how to accomplish the task. What mathematics content should be taught? How should it be taught? Who should teach it? Who should be expected to learn it? These specific questions reflect a larger question which continually resurfaces: Who determines which knowledge is core and common?

Education reform has always sparked controversy and mathematics education reform is no different. Tensions regarding mathematical content and the best way to teach and learn it sparked heated debates over “fuzzy mathematics” (Klein, 2003) in the New

Math era of the 1960s. Decades later, during the contentious Math Wars of the 1990s, a constructivist, process-oriented reform movement stimulated by the National Council of Teachers of Mathematics' (NCTM) *Curriculum and Evaluation Standards for School Mathematics* (1989), diverged from the traditional content-driven model emphasizing procedure and product. Schoenfeld (2004) asserts that reform's call for mathematical equity was perceived as a threat to excellence and the traditional stance that advanced mathematics can—and should—only be achieved by some.

All wars have causes and effects. Battles fought during the New Math era and the Math Wars that followed were no exception. These conflicts, “a microcosm of the larger historical ‘traditional versus progressive’ debate about American schooling” (Schoenfeld, 2004, p. 271), foreshadowed the current fight over a different set of standards. This fight reflects our long national history of diverse—and often opposing—beliefs about how to best educate children and who has the power to decide.

America's founders viewed public education as essential to the survival of our democracy, yet the United States Constitution does not mention education; the power to govern education resides with the states or the people (U.S. Const. amend. X). While education is traditionally a local affair, the federal government has expanded its educational role to protect civil rights and distribute aid in the last half-century. Recently, however, federal and private involvement in education reform has drastically increased (Ravitch, 2013). Conflicting perspectives about power in education mirror fundamental differences in broader politics: advocates for the freedom to oversee education at the state and local level seek to preserve Jefferson's system of little republics while proponents for

unified education goals promote Hamilton’s vision for a strong national presence. Who should govern education? The current debate over a new set of standards proves this 240-year-old question remains unanswered.

This researcher’s topic of interest is the CCSSM, which pair with English language arts/literacy (ELA) standards to comprise the Common Core State Standards (CCSS) (NGA & CCSSO, 2010). For the first time in our nation’s history, nearly all states and territories have adopted a common set of mathematics standards. To date, 42 states, the District of Columbia, four territories, and the Department of Defense Education Activity (DoDEA) are currently implementing the CCSSM. Puerto Rico and five states—Alaska, Nebraska, Texas, Virginia, and Minnesota—chose not to adopt the standards (though Minnesota did adopt the CCSSELA) and three states—Indiana, Oklahoma, and South Carolina—have recently dropped the CCSS (Academic Benchmarks, 2014; Strauss, 2014). In other states, politicians have threatened to stop using the CCSS or the outside assessments aligned to the CCSS, reflecting the volatility of the situation.

Previously, each state had its own education standards and proficiency levels, but the lack of consistency between states meant that a student’s educational experience could vary greatly, depending on geography. In 2009, the National Governors Association (NGA) Center for Best Practices and Council of Chief State School Officers (CCSSO) “recognized the value of consistent, real-world learning goals and launched this effort to ensure all students, regardless of where they live, are graduating high school prepared for college, career, and life” and led the effort to develop common literacy and

mathematics standards (NGA & CCSSO, 2010, “Development Process”). The CCSSM, which define a level of quality regarding what students should know and be able to do in mathematics, are researched-based, internationally benchmarked, and developed using the best existing state standards with involvement from teachers, content experts, state leaders, and the public (NGA & CCSSO, 2010, Development Process section, p. 1).

While the development of the CCSS was technically a state-led effort, the product is a de facto set of national standards which have generated both strong support and opposition.

Groups opposing the CCSS—both for English/language and mathematics—label it a national curriculum that infringes on the rights of local communities to make education decisions—prompting some governors to pull out of the CCSS by executive order (Strauss, 2014). Critics cite the lack of field testing, less-than-transparent development process, implementation challenges at the classroom level, the influx of privatized curriculum and assessment, and federal mandates to use standardized test data to evaluate teachers and students as reasons to withhold support (Ravitch, 2013). Even educators who generally support the standards, acknowledge challenges related to curricula alignment, teacher training, and the time required to meet higher expectations.

Proponents counter that national standards do not require a break with the longstanding American tradition of local control. The creation of national standards is not the “profound shift in the governance of education...national involvement has always been a part of education in the United States; total local control has never been the reality” (Jackson, 1994, p. v). Advocates also argue that national education standards will “ensure that our increasingly diverse and mobile population will have the shared

knowledge and values necessary to make our democracy work” (Smith, Fuhrman, & O’Day, 1994, p. 18). The national dialogue over national education standards continues.

Are the CCSS in general and the CCSSM in particular a federal takeover of education and a plot to destroy public schools? Are they a means to promote educational equity and prepare all learners for the 21st century? As the debate wages on, even education experts do not always find common ground when it comes to the Common Core.

Research Questions

It is within this contentious climate of American mathematics education that these research interests developed. Just as beliefs drive the creation of education policy—such as common, rigorous mathematics standards—teachers’ beliefs influence how they interpret and implement that policy. The complex relationship between beliefs and practice formed the foundation of this research which focused on the following questions:

- 1) *What do teachers believe about mathematics, teaching mathematics, and learning mathematics?*
- 2) *How do these teacher beliefs influence their interpretation and implementation of the Common Core State Standards for Mathematics?*

In research, “the questions that one chooses to ask and the data that one chooses to gather have a fundamental impact on the conclusions that can be drawn” (Schoenfeld, 2007, p. 70). Creswell (2009) identifies three research framework elements—philosophical and theoretical foundations, general strategies of inquiry, and detailed methods of data collection and analysis—to consider. The researcher analytically and

mathematically examined the topic and both research questions within this three-element framework. Mathematics can broadly be defined as the science of patterns. Early in the research process, patterns emerged among the three elements to reveal a common, overarching theme: the both-and perspective. To investigate a research problem existing within the complex system of American education, it logically followed that a uniquely American philosophical perspective—pragmatism—would yield the best opportunity to answer the research questions.

Philosophical Foundation

Educational and social reformer John Dewey (along with fellow Americans Charles Peirce and William James) opted for a pragmatic philosophical and practical approach to understanding how we think and learn, shaping his views on education and research (Cobb, 2007). In rejecting absolutism, Dewey defended pragmatism as the “systematic elaboration of the *logic and ethics of scientific inquiry*” (1933/2008, p. 24). Pragmatists reject the rigid dichotomy between post-positivism and constructivism, instead favoring an integrated worldview that accepts both the internal world existing within the mind and the external world existing independent of it. Instead of arguing over notions of reality, pragmatists take a real-world, practice-oriented stance, focusing on the problem at-hand and using a variety of tools to solve it (Creswell, 2009). By not committing to a singular reality, pragmatism “opens the door to multiple methods, different worldviews, and different assumptions, as well as to different forms of data collection and analysis” (Creswell, 2009, p. 11). Pragmatists view beliefs as the “suppositions and assumptions on which [people] risk acting” (Cobb, 2007, p. 11). The

concept of beliefs lies at the very heart of both what and how this researcher chose to investigate.

Theoretical Perspectives

The first theoretical theme that informed this research involves the relationship between teacher beliefs and practice. The theory that teachers' beliefs influence practice has been consistently demonstrated through extensive research (Philipp, 2007). The prevalence of this research topic supports McLeod's (1992) assertion that "all research in mathematics education can be strengthened if researchers will integrate affective issues into studies of cognition and instruction" (p. 575). This perspective justifies the importance of studying beliefs as a factor that impacts instruction and indicates the need to distinguish beliefs from other factors, such as knowledge in a general sense (Philipp, 2007) or specifically content-, pedagogical-, or curricular-knowledge (Shulman, 1986; Hill, Ball, & Schilling, 2008). After studying teachers' mathematical knowledge, Ernest (1989) suggested that beliefs may have a more powerful influence on teachers' actions than knowledge. Clearly, developing a greater understanding of such a powerful force in teaching is a worthwhile research endeavor.

Beliefs are "lenses that affect one's view of some aspect of the world" or "dispositions toward action" (Philipp, 2007, p. 259). Beliefs, therefore, drive action. Researchers have examined the relationship between what teachers believe about mathematics, learning mathematics, teaching mathematics (Ambrose, 2004; Ladson-Billings, 1999; Pajares, 1992; Philipp, 2007; Remillard & Bryans, 2004; Stigler in Spiegel, 2012), and mathematics standards (Zollman & Mason, 1992) for decades, yet not

all researchers define beliefs in the same way. Pajares (1992) cites the lack of clear definitions and poor conceptualization as major challenges that have plagued previous attempts at researching such a “messy construct” (p. 307). This study used Philipp’s (2007) definition of beliefs, as “psychologically held understandings, premises, or propositions about the world that are thought to be true” (p. 259). Well-designed research conducted since Pajares’ time provided a stable framework for understanding beliefs and how best to measure them. This researcher sought to contribute to the existing research base by investigating teachers’ beliefs and their practices of interpreting and implementing the relatively new phenomenon of the CCSSM.

There are many decisions and actions that take place in the teaching profession. The second theoretical theme that informed this research involves the practices of interpretation and implementation. As this inquiry related to teaching practices and the CCSSM, relevant literature included the ways teachers have interpreted and implemented previous educational reforms, policies, and standards. Beliefs, often resistant to change, act as a filter through which new phenomena are interpreted (Pajares, 1992). The way in which a phenomenon is interpreted will also impact the way in which it is implemented. Just as beliefs may be held with varying degrees of conviction (Philipp, 2007), implementation may be practiced with varying degrees of fidelity. While researching the effectiveness of reform curricula, Brown, Pitvorec, Ditto, & Kelso (2009) defined fidelity of implementation as the measure of faithfulness between something that is implemented and actions taken by the implementer. This study established a framework for understanding factors, including teacher beliefs, which influence the fidelity of

implementing curriculum. While the CCSSM is not a mathematics curriculum, this framework offers guidance to those studying how teacher beliefs influence the interpretation and implementation of mathematics standards. Teachers holding different beliefs about how children develop mathematical thinking may interpret and implement the same standard differently. The CCSSM is descriptive, not prescriptive; it dictates neither curriculum nor pedagogy (NGA & CCSSO, 2010) and is, therefore, open to multiple interpretations and wide variation in classroom implementation. This variation in practice raises questions and poses opportunities for educational research.

The view that beliefs about educational reform are related to its success or failure also informs this research—particularly significant in the current context of enacting the CCSSM. Previous studies of educational reform have indicated that the main obstacle to implementation was teachers' beliefs about mathematics teaching (Ross, McDougall, & Hogaboam-Gray, 2002; Ross, McDougall, Hogaboam-Gray, & LeSage, 2003). New knowledge about teaching and learning mathematics, including research-based evidence presented in the CCSSM, may not be powerful enough to change the underlying beliefs that teachers hold; this resistance to change may result in professional conflict and, ultimately, rejection of the reform. Examining the relationship between beliefs and practices related to educational reform may increase professional awareness that how teachers *feel* about a particular educational reform is closely related to what they *do* to contribute to its success or failure.

While educational reform can occur on an informal level, some reform is driven by formal changes in educational policy. Policy can be defined as a proposed or adopted

principle or course of action designed to influence decisions and actions to enable an organization to achieve long-term goals. In this sense, the CCSSM is an educational policy designed to influence the decisions and practices in the classroom in order to improve mathematics teaching and learning in the United States. Kingdon (2003) identified a four-step policy process—agenda setting, alternative specification, an authoritative choice, and implementation. This study focused primarily on the final step—implementation—and the ways beliefs influence how teachers interpret and implement the CCSSM. The Center for Education Policy recognizes the need to and importance of researching the CCSS and offers a compendium of studies related to the CCSS, including the topic areas of implementation, teacher preparation, and teacher professional development (Frizzell, 2014, “A Compendium”). To date, none of these studies specifically examined the relationship between teacher beliefs and practices related to the CCSSM—highlighting the need for further study.

There are many kinds of educational policy. Researchers who have investigated teacher beliefs and mathematics standards have generated findings most relevant to this inquiry. The CCSSM, by definition, is a set of mathematics standards that describes what students should know and be able to do at certain points in their educational journey from kindergarten through high school graduation. The CCSSM was created to ensure all high school graduates in the United States are college and career ready (NGA & CCSSO, 2010). While researchers have investigated other educational standards, including mathematics standards, no studies to date have focused on teacher beliefs and the CCSSM specifically, again indicating a research opportunity.

Research Design

The research topic and questions (asking both *what* and *how*) suggested a mixed methods design. A mixed methods researcher “tends to base knowledge claims on pragmatic grounds and assumes that collecting diverse types of data best provides an understanding of a research problem” (Creswell, 2009, p. 18). The relationship between beliefs and practice is a complex phenomenon existing within a complex system. The flexibility of a mixed methods design allowed the researcher to capture complexities with the freedom of a both-and research approach, utilizing: “both open- and closed-ended questions, both predetermined and emerging approaches,” and “both statistical and text analysis” (Creswell, 2009, pp. 18, 15) to yield different kinds of data that allow a more comprehensive picture to emerge (McMillan & Schumacher, 2010).

Just as research methodology reflects the values of a research community (Schoenfeld, 1994) it also reflects the values of the researcher. This mixed method design capitalized on the strengths and mitigated the inherent limitations of each approach. Statistics can easily be manipulated and misused, especially when providing information to an audience that does not fully understand quantitative data. Likewise, personal dispositions act as a filter through which data must pass, which may lead to inaccurate claims based on qualitative data. As both a “people person” and a “math person,” this researcher values both numerical data and the human stories behind them. McMillan & Schumacher (2010) assert that quantitative methods enable researchers to gather objective data that can be analyzed numerically while qualitative methods allow researchers to collect information directly from subjects, allowing for multiple

perspectives and narratives, to provide understandings for “how and why behavior occurs” (McMillan & Schumacher, 2010, p. 321). This blended methodology offered both “the structure of quantitative research and the flexibility of qualitative inquiry” (Creswell, 2009, p. 19).

This approach also suited the pragmatic purpose for the research. Beliefs influence how academic standards are interpreted and implemented. By integrating hard data supported by teacher voices, school districts can appreciate what teachers, individually and collectively, believe and how they enact these new mathematics standards. These research findings may help districts create appropriate, targeted professional development opportunities to improve teaching and learning under the CCSSM.

Not all mixed method designs are equal. Again, questions—and the sequence in which they are asked—determine the specific research design. An explanatory design uses “qualitative questions that provide explanations for findings from quantitative questions” (McMillan & Schumacher, 2010), first asking questions to generate statistical findings about *what* teachers believe, followed by qualitative questions to determine *how* those beliefs influence practice. The explanatory design “begins with a broad survey in order to generalize results to a population and then focuses, in a second phase, on detailed qualitative, open-ended interviews to collect detailed views from participants” (Creswell, 2009, p. 18). In this study, quantitative data was first gathered from a larger group of individuals to test the theory that not all teachers hold identical beliefs about mathematics, teaching mathematics, and learning mathematics. Then qualitative data was

collected from a smaller group of individuals, selected from the original sample, to explain the numerical data using teacher language and voices. This explanatory design yielded a database in which qualitative data helped the researcher develop a comprehensive understanding of the initial statistical results.

Significance

Philosopher and scientist Alfred Korzybski's (1933) quote, "the map is not the territory", clearly and concisely expresses this researcher's practical, theoretical, and philosophical understanding of learning, teaching, research, and the world in which we live. We must be aware that the representation of a thing \neq the thing itself. One person's map is never *the* map and no map will ever be a full-scale, corresponding match with the territory. If mathematics standards are the territory, then it logically follows that each teacher will construct, through interpretation, a personal and professional map to guide the implementation of those standards in the classroom. Researching this complex relationship between teacher beliefs and practices involving these recently adopted standards answers McLeod's (1992) call to investigate the role affect plays in interpreting and implementing educational policy—specifically, the CCSSM. Thus, affective findings increase awareness of the powerful influence of the beliefs teachers hold, inform those responsible for creating teacher professional development, and contribute knowledge to support the overarching goal of improving mathematics teaching and learning for all.

Conclusion

For the first time in history, nearly all children in the United States are learning under a common set of mathematics standards. Those *common* standards, however, are

affected by the beliefs of complex beings acting within a complex system. While humans are social beings who live and learn best in collaborative communities, the nuances of meaning and subtleties of perspective will always be unique to each individual. Cobb (2007) recommends mathematics education researchers, instead of adhering to one right and only perspective, should act as “bricoleurs” (p. 29) who construct meaning from a diverse range of available materials. Applying this approach to research afforded the opportunity to employ a pragmatic, both-and, mixed-method research bricolage—allowing multiple theories and tools to construct practical knowledge and answer the specific research questions about teachers’ mathematical beliefs and how those beliefs influence their interpretation and implementation of the CCSSM.

. This introductory chapter identifies the research questions and rationale for the study. As beliefs are found to be the best indicators of decisions and actions (Dewey, 1933/2008), it follows that teacher beliefs have a profound influence on their professional practice. This study examined teachers’ mathematical beliefs and their practices relating to the CCSSM. Chapter two reviews the literature that informs the study. Chapter three describes the research design and methods used to collect and analyze the data. Chapter four communicates the findings related to what teachers believe about mathematics, teaching mathematics, and learning mathematics and how those beliefs influence the professional practices of interpreting and implementing the CCSSM. Chapter five concludes with a summary of findings, the study’s limitations, and implications for the mathematics education community.

CHAPTER TWO: LITERATURE REVIEW

“Beliefs are the best indicators of the decisions individuals make throughout their lives...an assumption that can be traced to human beings’ earliest philosophical contemplations.” (Pajares, 1992, p. 307)

Introduction

This study addressed two research questions: *What do teachers believe about mathematics, teaching mathematics, and learning mathematics?* and *How do these beliefs influence their interpretation and implementation of the Common Core State Standards for Mathematics?* Beliefs are the bedrock and cornerstone at the heart of our actions (Corey, 1937) as well as the best indicators of the decisions individuals make throughout their lives (Dewey, 1910). If beliefs are mental representations of reality that guide thought and behavior (Pajares, 1992), then teacher beliefs are instrumental in defining pedagogical and content tasks and for processing information relevant to those tasks (Nespor, 1987).

This chapter reviews the literature on teacher beliefs and professional practice, in broad educational contexts and within the mathematics education field in particular, including interpretation and implementation of educational reform, policy, and standards. The review establishes the importance of studying affect in education yet acknowledges and describes challenges associated with investigating affective constructs and offers remedies to overcome them. Much progress has been made in the study of teacher beliefs, particularly in the field of mathematics education. Researchers offer a variety of theoretical interpretations of the construct as well as methodologies for studying them.

This review presents both analysis and synthesis of prior research efforts that inform this study. The chapter concludes by describing how this study seeks to extend knowledge in the mathematics education field by applying evidence-based theory and well-respected methods for examining teachers' mathematical beliefs and the ways they interpret and implement the relatively recent CCSSM.

Theoretical Literature on Teacher Beliefs

The mysterious nature of beliefs traditionally limited the subject to philosophical, theological, or spiritual inquiry. Because humans hold beliefs about nearly everything, narrowing the broad construct of beliefs is necessary to meet the pragmatic needs of any investigation. The study of beliefs in other professional fields, such as law and medicine, has resulted in a variety of meanings associated with the term, but also indicates that focusing on specific beliefs makes their “exploration feasible and useful to education” (Pajares, 1992, p. 308). This study investigated two research questions. First, what do teachers believe about mathematics, teaching mathematics, and learning mathematics? Second, how do those beliefs influence the ways in which teachers interpret and implement the CCSSM. Both topics central to the study, teacher beliefs and practices relating to interpretation and implementation, are informed by research conducted in the education and mathematics education fields.

Teacher Beliefs

The writings of M. Frank Pajares (broader education) and Randolph Philipp (mathematics education), building on an extensive amount of research on the subject, provide a theoretical framework for understanding and researching the relationship

between teacher beliefs and professional practice. Both Pajares (1992) and Philipp (2007) cite a wide array of researchers from all relevant fields, including Bandura (1986) and Dewey (1910), who find that beliefs are the best indicators of decisions and actions. Since beliefs influence thought and behavior (Pajares, 1992), teacher beliefs are instrumental in defining their professional practice. As an important factor in instruction, beliefs are a subject worthy of educational research. While a worthy, feasible, and useful pursuit, researching such a formidable construct presents considerable challenges.

Pajares (1992) cites the lack of clear definitions and poor conceptualization as major challenges that have plagued previous attempts at researching such a “messy construct” (p. 307). Fifteen years later, Philipp (2007) confirmed this finding, stating that researchers define beliefs in different ways (and some don’t define them at all!), leading to a lack of conventionality on the topic. Instead, due to the common use of the term *belief*, “many researchers have assumed that readers know what beliefs are” (Thompson, 1992, p. 129). To overcome inconsistencies and confusion in the field, it is important for researchers to distinguish beliefs from related concepts, clearly define the concept of beliefs, and—based on that definition—choose appropriate methodologies to study them.

Beliefs, Values, and Attitudes

Decades of research, both in other professional fields and within education, have resulted in a variety of meanings of the term *belief*. Pajares (1992) and Philipp (2007) denote the importance for researchers to clearly define and describe their meaning of the term. For the purposes of this study, this researcher used Philipp’s (2007) definition of beliefs, as “psychologically held understandings, premises, or propositions about the

world that are thought to be true” (p. 259). As “lenses through which one sees the world” (Philipp, 2007, p. 259), beliefs are a complex construct, closely related to values, attitudes, and knowledge. Researchers (Abelson, 1979; Nespor, 1987; Pajares, 1992; Philipp, 2007) not only assert the importance of defining beliefs, but also distinguishing them from these related affective components.

Values, which imply worth, “are associated with desirable/undesirable dichotomy” whereas beliefs “are associated with a true/false dichotomy” (Philipp, 2007, p. 259). A value can be thought of as a belief *in* something; in this sense values are a subset of beliefs. Philipp defines attitudes as “manners of acting, feeling, or thinking that show one’s disposition or opinion” (2007, p. 259). Attitudes are more adaptable than beliefs (Pajares, 1992). Even when presented with conflicting information, individuals may choose to interpret new findings in a way that supports a held belief. “The power of beliefs easily can outweigh the clearest and most convincing contrary evidence” (Munby, 1982). Some investigators caution against a purely static view of beliefs, opting to describe beliefs as “dynamic mental structures” that are *possible* to change through knowledge and experience (Thompson, 1992). However, there is general agreement that adult beliefs are more deeply engrained than attitudes and, therefore, are not as *easy* to change (Pajares, 1992).

Such distinctions, often subtle, are nonetheless helpful when clarifying the term for research purposes. Beliefs, values, and attitudes are so closely related, they can be thought of as threads that, when woven together, create the fabric of an entire belief system. Another component commonly associated with beliefs is knowledge. Researchers

who examine the relationship and distinction between knowledge and beliefs have yielded various, and sometimes conflicting, views.

Beliefs and Knowledge

Although there is general consensus that knowledge and beliefs are closely intertwined, the literature presents different theories regarding how (or whether) to separate the two constructs and determine which has a greater influence on action. From an epistemological perspective, knowledge can be defined as justified true beliefs. Philosophically, a belief may be an untrue proposition while knowledge must be not only true, but also grounded by evidence. Thompson (1992) proposed two distinctions between knowledge and beliefs: conviction and consensuality. Beliefs, unlike knowledge, can be held with varying degrees of conviction and knowledge, unlike beliefs, is consensual (Thompson, 1992, p. 130). The notion of having strong beliefs is a more familiar notion than knowing a fact strongly (Pajares, 1992). While it is widely accepted that people's beliefs may differ, knowledge typically requires general consensus resulting from agreed upon methods to prove or disprove the accuracy of a fact.

While it is clear that knowledge and beliefs are related, the exact nature of this complex relationship is not clearly understood. Some researchers claim that knowledge, though influenced by beliefs, offers greater insight into human behavior or action (Roehler, Duffy, Herrmann, Conley, & Johnson, 1988 as cited in Pajares, 1992, p. 312). Those who have studied teacher beliefs and knowledge have gone on to distinguish different types of knowledge teachers use to inform their decisions and actions in the classroom.

Researchers have found that, in addition to educational and mathematical beliefs, knowledge also impacts a teacher's professional practice. Some study different kinds of knowledge used in teaching, such as content-, pedagogical-, or curricular-knowledge (Shulman, 1986; Hill et al., 2008). Others delve beyond mere knowledge to examine knowing-about (consisting of knowing-that, knowing-how, and knowing-why) which is usually the focus of formal education, compared with knowing-to, which requires awareness and depends on the "structure of attention in the moment" (Mason & Spence, 1999, p. 135). Teaching is a cognitively demanding profession (Ambrose, 2004) which requires thinking about a variety of things, called knowledge in use (Ball, 2000), while helping children learn.

Some in the education field question the need to distinguish beliefs from knowledge at all. Lewis (1990) argued that, since beliefs are the origin of all knowledge, the two constructs are synonymous. Other researchers disagree with Lewis's (1990) view that knowledge and beliefs are synonymous, indicating the need not only to distinguish beliefs from knowledge in both a general (Philipp, 2007) or specific (Hill et al., 2008; Shulman, 1986) sense, but also to determine which is more influential.

Many researchers (Shulman, 1986; Nespor, 1987; Ernest, 1989; Pajares, 1992, Philipp, 2007; Hill et al., 2008) propose that beliefs—not knowledge—are the most powerful force in determining teacher decisions and actions in the classroom. When compared to knowledge, Nespor (1987) found beliefs are not bound by requirements of logic, reason, consensus, or consistency, are more rigid, and less open to examination and evaluation. Therefore, "beliefs are far more influential than knowledge

in determining how individuals organize and define tasks and problems and are stronger predictors of behavior (Nespor, 1987, p. 321). Philipp asserts this “debate about the relationship between knowledge and beliefs is unlike to cease” and advocates that researchers instead “take a clear stance on how they are viewing beliefs” (2007, p. 267).

For the purposes of this study, the researcher acknowledged a variety of complex and connected factors, including knowledge, that affect how teachers teach. Yet, this researcher proposes teachers’ beliefs about mathematics, teaching mathematics, and learning mathematics play the more significant role in how they interpret and implement the CCSSM and therefore examined what teachers believe, not what they know. This view is strongly supported by the work of numerous researchers, including Ernest (1989) who found that mathematics teachers with similar knowledge teach in very different ways, suggesting that beliefs may be more useful in understanding the decisions teachers make. Pajares (1992) continues this view with a discussion on the filtering effect of beliefs. The affective, evaluative, and episodic nature of beliefs makes them a filter through which new phenomena are interpreted; beliefs influence the processing of information and shape subsequent thinking (Abelson, 1979; Ernest, 1989; Nespor, 1987; Rokeach, 1968). Again, Philipp’s (2007) describes beliefs “as lenses that affect one’s view of some aspect of the world or as dispositions toward action” (p. 259). As lenses, beliefs determine how teachers *interpret* the CCSSM; as dispositions toward action, beliefs influence how teachers *implement* the CCSSM.

Teacher Beliefs and Practice

The relationship between teacher beliefs and professional practice has been a very popular subject of educational research, especially in mathematics education (Philipp, 2007), and was the primary theme of this inquiry. To date, a quick search using the terms *teacher*, *beliefs*, and *mathematics* reveals more than 300,000 hits on Google Scholar. Clearly, the topic is of interest to those in mathematics education. The prevalence of this research topic supports McLeod's (1992) proposition that mathematics education research can be strengthened by investigating affective issues (p. 575).

This study was guided by the theoretical perspective that teacher beliefs drive professional decisions and actions. The theory that teacher beliefs influence practice has been consistently demonstrated through extensive research in education (Philipp, 2007). In mathematics education, the finding that teachers' beliefs about teaching and learning mathematics significantly affect the form and type of instruction they deliver has been well-established (Clark & Peterson, 1986; Vacc & Bright, 1999). There is significant evidence to support the view that teaching behavior is profoundly or subtly influenced by what teachers believe mathematics should be (Fullan, 1982; Thompson, 1984). Research findings also indicate that some teacher beliefs about mathematics lead to inappropriate teaching practices (Ferrini-Mundy, 1986) and may limit what students can learn (Bauch, 1984). A multitude of well-designed studies provide a variety of frameworks for examining the beliefs-practice relationship.

Mathematics education researchers often specialize their inquiry, focusing on teacher beliefs in relation to specific aspects of their practice or certain populations of the

profession (i.e. in-service or pre-service teachers or elementary vs. secondary teachers).

Philipp (2007, p. 281) identifies four common areas of research on teacher beliefs:

1. students' mathematical thinking
2. curriculum
3. technology
4. gender

Findings from the first two areas are most relevant to this study, offering perspectives on the relationship between beliefs and practice, practical approaches to studying the construct, and methods to generate useful data.

Some researchers, adopting a more dynamic view of beliefs, examine how beliefs can be changed by vivid experiences with children (Ambrose, 2004), using children's mathematical thinking (Fennema, Carpenter, & Loef, 1990), and professional development opportunities (Fennema, et al., 1996; Peterson, Fennema, Carpenter, & Loef, 1989). Others, particularly those studying pre-service teachers, find that beliefs are highly resistant to change. Pre-service teachers hold strong beliefs about teaching and learning in general (Holt-Reynolds, 1992) and specifically about teaching and learning mathematics (Ball, 1990; McDiarmid, 1990), which are difficult to change—even when they learn new theories and concepts as part of their teacher education program (Kagan, 1992). Instead, pre-service teachers' beliefs are based on previous vivid episodes and events (Pajares, 1992) or their own learning experiences as students (Ball, 1990; Holt-Reynolds, 1992; Knowles & Holt-Reynolds, 1991). These findings, from research focused on a specific population or aspect of mathematical beliefs, indicate that while it

is *possible* to change teacher beliefs, it is not an *easy* task—again, supporting the view that new knowledge or experiences may not be sufficient to change beliefs.

Researchers who adhere to a more stable view of the beliefs teachers hold, which impact their professional decisions and actions, focus on the sources of such strong beliefs. The recognition that culture plays a role in how individuals and societies form beliefs have lead some investigators to focus on the cultural sources of beliefs that influence teaching and learning mathematics (Stigler & Hiebert, 1998; Stigler in Spiegel, 2012). Others emphasize sub-cultural factors—such as race or gender—that influence mathematical beliefs held by students and teachers, or relate to mathematics educational equity (Boaler, 2002; Fennema, Carpenter, Jacobs, Franke, & Levi, 1998; Tartre & Fennema, 1995). While the thousands of studies available on teacher beliefs and practice differ in terms of specific topics and methodologies, all share a common theme: the research agenda of each investigation requires researchers to adopt a particular definition of beliefs relating to specific teaching practices to focus the investigation and render the study of such a complex subject possible.

As there are many practices that comprise the teaching profession, the second major theoretical theme that informed this research involves the specific practices of interpretation and implementation. This study focused specifically on teacher interpretation and implementation of the CCSSM—a document of mathematics education reform, policy, and standards.

Theoretical Literature on Interpretation and Implementation

Teacher Beliefs and Mathematics Education Reform

Beliefs, often resistant to change, act as a filter through which new phenomena are interpreted (Pajares, 1992). The way in which a phenomenon is interpreted will also impact the way in which it is implemented. The view that teacher beliefs influence the interpretation and implementation of educational reform (impacting its success or failure) also informs this research—particularly significant in the current context of enacting the CCSSM. Lack of teacher training and inadequate mathematics content knowledge have been cited as contributing factors to previous failed mathematics education reform efforts, particularly in the New Math of the 1950s and 1960s (Klein, 2003; Schoenfeld, 2004). The well-demonstrated connection between beliefs and knowledge and the powerful effect of beliefs on instruction, as indicated in the previous section, led many researchers to study teacher beliefs in the context of mathematics education reform movements.

A main obstacle to implementation of reform is teachers' beliefs about mathematics teaching (Ross et al., 2002). This theory was particularly relevant in that new knowledge about teaching and learning mathematics, including research-based evidence presented in the CCSSM or in professional development classes, may not be powerful enough to change the underlying beliefs that teachers hold; this resistance to change may result in professional conflict and, ultimately, rejection of this latest educational reform.

Education reform, whether during the New Math era of the 1950s and 1960s, the *Curriculum and Evaluation Standards for School Mathematics (Standards)* (NCTM, 1989) reform movement during the 1990s, or the current implementation of the CCSSM, affords researchers the opportunity to examine teachers' mathematical beliefs and practices during a transition. The original NCTM *Standards* (1989) called for changes in mathematics classrooms across the nation to meet diverse learner needs and an increased demand for a mathematically literate workforce. Later NCTM publications, including the *Professional Standards for Teaching School Mathematics* (1991), the *Assessment Standards for School Mathematics* (1995), and the updated *Principles and Standards for School Mathematics* (2000), sustained mathematics education reform through the end of the twentieth century and laid the groundwork for the eventual creation of the CCSSM. The NCTM-inspired reform affected curriculum development and challenged educators to alter traditional models of teaching and learning. These curricular and pedagogical changes led to hundreds of studies about mathematics education reform (Philipp, 2007).

Researchers examined how teaching in a reform mathematics classroom might challenge beliefs about teaching efficacy and the way children learn mathematics. The reform teacher serves as an orchestrator of mathematically meaningful discourse rather than sole knowledge source (Stein, Engle, Smith, & Hughs, 2008). Within this non-hierarchal structure, learners construct knowledge through complex, non-linear interactions with the teacher and other students. Similarly, in the constructivist reform classroom, teachers “no longer present the content through clear demonstrations; they must instead create the conditions that will allow students to take their own effective

mathematical actions” (Smith, 1996, p. 393). Smith (1996) proposed that most teachers (as well other adults and students) in the United States hold a core set of beliefs about the nature of school mathematics that influence their sense of teaching efficacy and are comprised of four commitments:

1. mathematical content
2. teaching mathematics
3. learning mathematics
4. mathematical authority (pp. 390-391).

These core beliefs, most often reflecting a traditional view of mathematics and a teaching-by-telling model of instruction, were challenged during the mathematics education reform era of the 1990s. Smith (1996) also found:

Mathematics teachers, both practicing and prospective, respond to the reforms in different ways: from ignoring, downplaying, or openly resisting the changes; to retaining the telling model under the cover of reform; to embracing the heart of the reform, struggling to change, but falling short; to managing deep changes, and achieving new levels of success. (p. 395)

While many researchers, including Smith, focus on changing teacher beliefs to effect substantial change, their work also confirms the view that the core beliefs teachers hold—about content and the way teaching and learning occur—influence their teaching decisions and actions related to education reform. Examining the relationship between beliefs and practices may increase professional awareness that what teachers *believe* about the content and pedagogy related to a particular educational reform influences what

they *do* to promote its success or failure.

Meaningful parallels exist between the educational reform movements of the 1990s and today. The core mathematical beliefs underlying the CCSSM are consistent with the constructivist beliefs underlying the *Standards*-based reform. Smith's (1996) four commitments provide a useful framework for identifying underlying beliefs reflected in the CCSSM reform period. In terms of mathematics content, the CCSSM views mathematics as no longer simply a fixed collection of isolated facts and procedures; equally important is mathematical understanding—the “ability to justify...*why* a particular mathematical statement is true or where a mathematical rule comes from” (NGA & CCSSO, 2010, p. 4). Similarly, while the structure of the discipline must shape mathematics standards, they should not be the only consideration. The CCSSM (NGA & CCSSO, 2010) states:

the sequence of topics and performances that is outlined in a body of mathematics standards must also respect what is known about how students learn....In recognition of this, the development of these Standards began with research-based learning progressions detailing what is known today about how students' mathematical knowledge, skill, and understanding develop over time. (p. 4)

Concerning mathematics teaching and learning, the CCSSM (NGA & CCSSO, 2010) lists eight Standards for Mathematical Practice:

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.

3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning. (pp. 6-8)

The first of these practice standards are based on the 1989 NCTM process standards: problem solving, reasoning and proof, communication, representation, and connections while the rest relate to the strands of mathematical proficiency identified in the National Research Council's (NRC) *Adding It Up* (2001): adaptive reasoning, strategic competence, conceptual understanding, procedural fluency, and productive disposition (NGA & CCSSO, 2010, p. 6). These Standards for Mathematical Practice, therefore, reflect the same constructivist approach called for in earlier *Standards*-based reforms. The importance of connecting concepts with skills as well as understanding with procedures is hallmark to both reform movements. Finally, the CCSSM reflects earlier reform efforts to reconsider mathematical authority by requiring students to check "answers to problems using a different method" and "continually ask themselves, 'Does this make sense?'" They can understand the approaches of others to solving complex problems and identify correspondences between different approaches" (NGA & CCSSO, 2010, p. 7). Clearly, the mathematical beliefs teachers held influenced their reaction to earlier reform movements; it is likely, therefore, that the mathematical beliefs held by

today's teachers will also impact how they respond to similar changes during the CCSSM era of reform.

The *Standards*-based reform movement “took the nation by storm” and shaped the writing not only of state mathematics standards but also most curricula published in the 1990s (Schoenfeld, 2004, p. 269). These changes led many researchers to examine the connection between teacher beliefs and interpretation and implementation of reform curricula. Just as beliefs may be held with varying degrees of conviction (Philipp, 2007), implementation may be practiced with varying degrees of fidelity.

While researching the effectiveness of reform curricula, Brown et al. (2009) defined fidelity of implementation as the measure of faithfulness between something that is implemented and actions taken by the implementer. This study established a framework for understanding factors, including teacher beliefs, which influence the fidelity of implementing curriculum. Brown et al. (2009) determined that there was considerable variation in the enactment of the curriculum, particularly regarding the level of fidelity to the curriculum authors' intended lesson (the beliefs underlying the curriculum). Fidelity to the curriculum, therefore, was a focus of professional development. A different perspective of professional development, offered by Franke, Carpenter, Levi, and Fennema (2001), suggests that sustained improvement in practice comes not from training teachers to simply remain faithful to a given curriculum, but rather to be reflective practitioners who continually evaluate and adjust their practices in response to student needs (p. 658). Most research on fidelity of implementation, including these two studies, use classroom observations to examine teacher's decisions

and actions. While some researchers investigate how beliefs (among other factors) might affect the implementation of a curriculum, others elect to study how using a curriculum might affect teacher beliefs.

Collopy (2003) attempted to investigate changes in elementary school teacher beliefs (about teaching and learning mathematics) and instructional practices after using the reform curriculum *Investigations in Number, Data, and Space*. The two teachers in the study were similar in background (knowledge, training, and experience), teaching, situation, and willingness to pilot the reform curriculum. Their vast differences in enacting the program (one successful, the other not) was attributed to differences in their beliefs about mathematics and how best to teach it and whether these beliefs were compatible with the beliefs underlying the *Investigations* curriculum (Collopy, 2003). Remillard and Bryans (2004) continued this course of study, presenting the construct *orientation toward curriculum* and a model to reflect the integrated relationship among teacher perspectives and beliefs, curriculum, and teacher learning. Research in the 2000s extended and supported the findings from research conducted the previous decade: if teachers' beliefs are compatible with the underlying philosophy and materials of a curriculum, there is greater likelihood that the curriculum will be fully implemented (Hollingsworth, 1989; Richardson, 1990).

For the purposes of this study, it is important to note the CCSSM are descriptive, not prescriptive. They dictate neither curriculum nor pedagogy (NGA & CCSSO, 2010) and are, therefore, perhaps even more open to multiple interpretations and wide variation in classroom implementation. For example, the CCSSM calls for fourth graders to

“develop fluency with efficient procedures for multiplying [multi-digit] whole numbers; understand and explain why the procedures work based on place value and properties of operations; and use them to solve problems” (NGA & CCSSO, 2010, p. 27). Teachers holding different beliefs about how children develop mathematical thinking may interpret and implement this standard differently. Teacher A, who holds more traditional mathematical beliefs, may explicitly teach several algorithms to students via direct instruction and provide opportunities for students to practice using the modeled algorithms to increase proficiency with each. Teacher B, however, who holds more constructivist beliefs, may provide challenging word problems that students solve (by inventing their own algorithms) and then encourage students to communicate, compare, and contrast various strategies through classroom discourse.

The overarching goal of mathematics education is to help all children become mathematically proficient. The NRC’s *Adding It Up: Helping Add Children Learn Mathematics* (2001) stated that the teaching and learning of school mathematics should be guided by an “integrated and balanced development of all five strands of mathematical proficiency,” which include:

- 1) conceptual understanding
- 2) procedural fluency
- 3) strategic competence
- 4) adaptive reasoning
- 5) productive disposition (p. 11)

The NRC warned against instruction based on “extreme positions that students learn, on one hand, solely by internalizing what a teacher or book says or, on the other hand, solely by inventing mathematics on their own” and identified “one of the most serious and persistent problems facing school mathematics in the United States is the tendency to concentrate on one strand of proficiency to the exclusion of the rest” (2001, p. 11). Teachers who hold very strong and opposing mathematical beliefs may focus their attention on different proficiency strands in the classroom.

The beliefs underlying the CCSSM are constructivist in nature, reflected in content, research-base, and the eight Standards for Mathematical Practice (NGA & CCSSO, 2010). This balanced conception of constructivism allows for invention as well as teaching and does not ignore basic facts or computation procedures, but rather builds proficiency by integrating skills with conceptual understanding and problem solving. The consonance or dissonance between a teacher’s beliefs (about mathematics, teaching, and learning) and the beliefs that underlie the CCSSM may explain how the same standards look different in the hands of each teacher. This variation in practice, related to the filtering effect of beliefs, raises questions and poses opportunities for educational research.

Research on the Beliefs-Practice Relationship in Mathematics Education.

While there exists a vast amount of research on teacher beliefs and mathematics education reform, of particular significance to this study was a collection of research on the beliefs-practice relationship which evolved over decades, beginning in the 1980s, at the University of Wisconsin-Madison. Early studies in teachers’ mathematical attitudes

(Fennema, Carpenter, & Peterson, 1987) and using children's mathematical thinking in classroom teaching (Carpenter, Fennema, Peterson, Chiang, & Loef, 1988) generated a line of research on mathematical beliefs and practices that included the development of a survey instrument to assess such beliefs (Fennema et al., 1990) and a pedagogical approach known as Cognitively Guided Instruction. Several findings emanated from this generation of mathematics education research.

At the heart of many of these studies was the beliefs-practice relationship in mathematics education. Fennema et al. (1996) found that the beliefs-practice relationship is complex and teachers, like students, learn and change differently. There was variation among the teachers in the study regarding how much change in beliefs and instruction occurred, when the changes occurred, and the order in which they occurred (Fennema et al., 1996, p. 429). For some teachers, changes in beliefs preceded changes in practice. For others, changes in beliefs followed changes in practice. For a third set of teachers, changes in beliefs and practice appeared to progress concurrently. Fennema et al. (1996) concluded that when teachers began to see their students learning in a manner consistent with their training on student thinking, there began to be "iterative changes in teachers' knowledge, instruction, and beliefs" (p. 431). While this study suggested that some teachers made rapid (within one year) changes in beliefs and/or practices, other research following this line indicated a much slower change in mathematical beliefs.

As more teachers participated in Cognitively Guided Instruction programs, more opportunities arose for researchers to examine changes in teacher beliefs and practices. One such study, designed to capture teachers' generative change, found teacher beliefs

were most likely to change through participation in supportive professional communities and opportunities to engage in practical inquiry with students in the classroom (Franke, et al., 2001). These same researchers asserted that awareness of beliefs and actions enable teachers to generate their own learning (Franke, et al., 2001). While the findings of these studies contributed knowledge about beliefs of in-service teachers, a similar line of research examined the beliefs of pre-service teachers.

Ambrose (2004) stated that beliefs about teaching and learning mathematics are part of a larger system of beliefs prospective teachers hold about teaching in general (p. 6). Ambrose, citing Pajares' recognition of beliefs as emotion-based (1992), found that experiences working closely with children in the classroom, combined with a mathematics content course and guided reflection, promoted changes in prospective teachers' mathematical beliefs (2004, p. 92). Ambrose's work led to the development of alternate tools to assess the complex phenomenon of teacher beliefs (Ambrose, Philipp, Chauvot, & Clement, 2003; Ambrose, Clement, Philipp, & Chauvot, 2004).

Regardless of the direction of influence (beliefs-practice or practice-beliefs), this section of the review provides historical context and evidence to support the theoretical perspective of this research study: that teacher beliefs about mathematics, teaching mathematics, and learning mathematics play a powerful role in the ways teachers interpret and implement educational reform. The conclusion that effecting change in professional practice evolves over several years and requires continual opportunities for professional development and support (Fennema et al., 1996) was particularly relevant to this research. The teacher participants in this study were in the fourth year of CCSSM

implementation; it was the ideal time to examine teachers' decisions and actions pertaining to these relatively new mathematics standards. Also, while many of these studies examined changes in teacher beliefs, these works offered frameworks and approaches for how to study beliefs or practices at a given point in time. Many of the studies combined the survey (either Likert-type or alternative) method with qualitative approaches such as classroom observation or interviewing teachers. Taken collectively, they informed this study of how teacher beliefs influence the interpretation and implementation of the CCSSM which also serves as a document of educational policy.

Teacher Beliefs and Educational Policy

Another set of theories that informed this research involved teacher beliefs and educational policy. Policy can be defined as a proposed or adopted principle or course of action designed to influence decisions and actions to enable an organization to achieve long-term goals. In this sense, the CCSSM is educational policy designed to influence the decisions and practices in the classroom in order to improve mathematics teaching and learning in the United States. Kingdon (2003) identified a four-step policy process—agenda setting, alternative specification, an authoritative choice, and implementation. For the vast majority of states and territories in the United States, the first three stages of the CCSSM policy process have already occurred. The second part of this study involved the final policy step—implementation—of the CCSSM.

The Center for Education Policy recognizes the need for and importance of research on the CCSS (encompassing both English/language arts and mathematics) and offers a compendium of studies related to the CCSS, including the topic areas of

implementation, teacher preparation, and teacher professional development (Frizzell, 2014, A Compendium section). Just as a wealth of studies followed the *Standards*-based reform movement in the 1990s and 2000s, a similar research boom occurred following the passing of the No Child Left Behind (NCLB) (2002) policy. It is reasonable to anticipate a similar pattern in the coming decades of research on mathematics education associated with the widespread adoption of the CCSSM in 2010. Likewise, findings that teacher beliefs influenced the enactment of earlier mathematics education policy indicate the likelihood that teacher beliefs—with their filtering effect on interpretation—will also play a powerful role in the process of CCSSM implementation. While research on the CCSSM is growing, to date, none of these studies specifically examined the relationship between teacher beliefs and practices of interpretation and implementation of the CCSSM—highlighting the need for further study.

Teacher Beliefs and Mathematics Standards

Most closely related to this inquiry were studies that have examined the relationship between teacher beliefs and mathematics standards. While also a document of educational reform and policy, the CCSSM, by definition, is a set of mathematics standards that describe what students should know and be able to do at certain points in their educational journey from kindergarten through high school graduation and were created, in part, to ensure high school graduates in the United States are college and career ready (NGA & CCSSO, 2010).

The CCSS (both English language arts/literacy and mathematics) began in 2007 during a meeting of the NGA and the Council of Chief State School Officers (CCSSO)

with the intent to bridge the gap between uniform national (not federal) standards and local control in education. Adoption of the CCSS is voluntary. While federal funding *is* tied to having standards, states may choose to adopt the CCSS (in either or both content area) *or* other rigorous standards (NGA & CCSSO, 2010). Local control and national standards, therefore, need not be mutually exclusive. However, given the financial challenges of most state and local educational organizations, the comingling of acceptance of federal Race to the Top (2009) funding with the expectation to adopt the CCSS has resulted in a common misconception that the federal government was involved in the creation of the Common Core.

The CCSSM represents progress toward national mathematics standards. While the widespread adoption of the *same* set of K-12 mathematics is unprecedented in the United States, it is not the first time in our nation's history that we have had a national model for mathematics standards. Even a brief foray into the recent history of mathematics standards in the United States can provide a more comprehensive understanding of the CCSSM. Research on teacher beliefs and an earlier set of mathematics standards provides insight into how best to examine the relationship between teacher beliefs and practices associated with standards.

In April 1983, the National Commission on Excellence in Education released the report *A Nation at Risk* which warned

The educational foundations of our society are presently being eroded by a rising tide of mediocrity that threatens our very future as a Nation and a people....If an unfriendly foreign power had attempted to impose on

America the mediocre educational performance that exists today, we might well have viewed it as an act of war. (NCEE, 1983, p. 1)

Education became, once again, part of the national agenda. Klein (2003) acknowledges “the timing for the NCTM *Standards* could not have been better. The nation was looking for benchmarks that could improve education....by default they became the national model for standards” (The NCTM Standards section, para. 13). They were readily endorsed by a variety of organizations and, in less than a decade, most state standards aligned with those created by the NCTM (Klein, 2003, The NCTM Standards section, para. 14).

The *Standards* encouraged a complete shift in mathematics education in the United States—a change in the underlying beliefs about what mathematics is, how best to teach and learn it, and who should pursue it as a course of study. The constructivist, process-oriented reform movement diverged from the traditional content-driven model emphasizing skill and procedure, and called for a commitment to mathematics educational equity, sparking pedagogical, content, and curricular changes that would lead to a battle known as the Math Wars (Schoenfeld, 2004). The *Standards* (1989) presented an opportunity for researchers to examine the consonance or dissonance between teacher beliefs and the beliefs underlying the standards themselves.

In researching teacher beliefs about the *Standards*, Zollman and Mason (1992) developed a survey that used language directly taken from (or an inverse of language from) the *Standards* and found that people’s stated beliefs are different than their actions—presenting a substantial obstacle in implementation of both standards and

curriculum. While Zollman and Mason (1992) were measuring the alignment (or misalignment) of teacher beliefs and actions relating to the NCTM *Standards*, their work offered guidance regarding approaches and methodologies for examining this relationship associated with the implementation of a new set of mathematics standards.

Reflecting human nature, teachers may hold a wide variety of beliefs associated with the CCSSM. For example, beliefs may be political (whether the CCSSM represents a loss of local control over education), practical (relating to curriculum materials), or organizational (the need for teacher training and support). Heeding the advice to narrow the focus of investigation to specific beliefs *about* a certain area of interest, this study focused on beliefs related to the actual standards themselves; what teachers believe about mathematics, teaching mathematics, and learning mathematics and how those beliefs influence their decisions and actions associated with the CCSSM.

Synthesis of Theoretical Literature on Teacher Beliefs, Interpretation, and Implementation

Teachers possess beliefs about education—instruction, students, learning, and content—and their professional practice is guided by these beliefs. Teacher beliefs are instrumental in defining pedagogical and content tasks (Nespor, 1987). Although the study of teacher beliefs is a valuable part of mathematics educational research, this area of inquiry presents challenges to researchers, including poor conceptualization, inconsistent definitions, and different understandings and uses of the word *beliefs*. The literature presents extensive findings that establish a theoretical framework for investigating the relationship teacher beliefs and the practices of interpretation and

implementation. Based on Pajares' (1992) synthesis of research on the subject, this study was based on the following six "fundamental, reasonable assumptions" about the nature of teacher beliefs:

- 1) Beliefs are formed early and tend to self-perpetuate, persevering even against contradictions caused by reason, time, schooling, or experience.
- 2) The belief system has an adequate function in helping individuals define and understand the world and themselves.
- 3) Knowledge and beliefs are inextricably intertwined, but the potent affective, evaluative, and episodic nature of beliefs makes them a filter through which new phenomena are interpreted.
- 4) Thought processes may well be precursors to and creators of belief, but the filtering effect of belief structures ultimately screens, redefines, distorts, or reshapes subsequent thinking and information processing.
- 5) Beliefs are instrumental in defining tasks and selecting the cognitive tools with which to interpret, plan, and make decisions regarding such tasks; hence, they play a critical role in defining behavior and organizing knowledge and information.
- 6) Individuals' beliefs strongly affect their behavior. (pp. 324-325)

In light of these assumptions, researching teacher beliefs is a valuable path of educational inquiry. Taken as a whole, the findings indicate a "strong relationship between teachers' educational beliefs and their planning, instructional decisions, and

classroom practices” (Pajares, 1992, p. 326). Although researching such a complex construct presents challenges, investigators can clear the path of inquiry.

When they are clearly conceptualized, when their key assumptions are examined, when precise meanings are consistently understood and adhered to, and when specific beliefs constructs are properly assessed and investigated, beliefs can be the single most important construct in educational research” (Fenstermacher, 1979 as cited in Pajares, 1992, p. 329).

To this point, the described literature provides a theoretical framework for establishing a clear conceptualization of teacher beliefs, determining a precise meaning for the term, and building support for the fundamental assumptions adopted in the study. Attention now shifts to literature informing the selection of appropriate methodologies and instruments to generate meaningful data that specifically relates to teacher beliefs (about mathematics, teaching mathematics, and learning mathematics) and their practices associated with interpreting and implementing the CCSSM.

Methodological Literature on Teacher Beliefs and Practice

Just as researchers must clearly define and conceptualize beliefs, they must also develop and use carefully chosen models and instruments to study the relationship between teacher beliefs and professional practice (Munby, 1982). Mathematics education researchers typically study teacher beliefs using one of two approaches: the case-study method or a beliefs-assessment instrument (Philipp, 2007). Over the course of the past several decades, the methods used to study teacher beliefs and other affective

components of educational practice have changed, allowing more diversity, flexibility, and opportunity.

Due to the position that beliefs must be inferred (Pajares, 1992), most educational research conducted prior to 1992 was “interpretive in nature, employed qualitative methods of analysis, and was comprised of in-depth case studies with small numbers of subjects” (Philipp, 2007, p. 262). Many researchers suggested more in-depth case studies were needed to build a more comprehensive understanding of teacher beliefs (Thompson, 1992).

Case Studies

Creswell (2009) describes a case study as an in-depth exploration of a bounded system using various data collection procedures over a period of time (p. 227). The ability to infer teacher beliefs may require such in-depth exploration; multiple data sets may be triangulated for cross-validation. While the case method approach offers researchers the opportunity to gather rich data sets from which to generate a theory related to teacher beliefs, this approach is not practical when measuring the beliefs of large populations in order to test theory. Instead, researchers may elect to develop alternative means to measure teacher beliefs on a grander scale

Over the last twenty years, however, many researchers have sought a different approach, developing instruments to quantitatively measure teacher beliefs related to children’s mathematical thinking (Fennema et al., 1990), mathematics education reform (Lloyd & Wilson, 1998; Sztajn, 2003), and mathematics standards (Zollman & Mason, 1992). To mitigate the limitations of earlier studies, subsequent researchers have chosen

to modify original instruments to suit their research agenda (Capraro, 2001) or use technology to create alternative, more sensitive instruments than the typical Likert scale survey (Ambrose, et al., 2003).

Likert Scale Instruments

One of the most recognized instruments designed and used to measure teacher beliefs in mathematics education is the Mathematics Beliefs Scales (MBS) survey (Fennema et al., 1990). This survey was adapted from an earlier version (Fennema et al., 1987), developed under a NSF grant at the University of Wisconsin, Madison. The MBS is a paper-and-pencil Likert-type instrument that contains 48 statements. Responses range from: A =Strongly Agree, B=Agree, C =Undecided, D= Disagree, E= Strongly Disagree where A=5, B=4, C= 3, D= 2, E= 1. The survey was coded as follows: positive items were left as-is and negative items (5, 7, 8, 11, 14, 15, 16, 17, 18, 22, 23, 24, 26, 29, 34, 35, 38, 39, 42, 44, 45, 46, 47, and 48) were coded in the opposite direction. The responses were added to get a total for each teacher, and a mean score was obtained by dividing by 48. Overall mean scores reflected whether teachers' mathematical beliefs were consistent with a low or high constructivist view of teaching and learning mathematics.

Fennema et al. (1990) created items reflecting four sub-scales, or categories:

- Role of the Learner
- Relationship Between Skills and Understanding
- Sequencing of Topics
- Role of the Teacher

The Role of the Learner scale contained items relating to how children learn mathematics; a high score indicated a constructivist view of learning and a low score indicated a traditional view (learning via direct instruction). The Relationship between Skills and Understanding measured whether a teacher believed skills should be taught in relation to conceptual understanding and problem solving; a high score suggested an integrated approach while a low score suggested a view of skills as isolated from understanding and application. Scale 3, the Sequencing of Topics, assessed teacher beliefs about the sequencing of topics in addition and subtraction instruction; a high score indicated a belief that children's development of mathematical ideas should inform instructional sequence while low score indicated belief that instructional topics should be based on the formal structure of mathematics. The Role of the Teacher, the fourth and final scale, assessed teacher beliefs about how mathematics (addition and subtraction) should be taught; a high score reflected a belief that mathematics instruction should facilitate children's construction of mathematical knowledge while a low score reflected a belief that instruction should be organized to facilitate teacher's presentation of knowledge (Fennema et al., 1990).

Fennema et al. (1990) developed the MBS, adapted from an earlier version, to assess teachers' mathematical beliefs and determine whether training in and use of the Cognitively Guided Instruction (CGI) research-based professional development program, designed to use children's mathematical thinking to inform mathematics instruction, changed teachers' beliefs over time. Researchers administered the MBS to both CGI and control teachers as a pretest (before CGI training) and posttest (after one year of using

CGI in the classroom). Using Cronbach's alpha, on a sample of 39 teachers, the "internal consistency of teachers' scores on the total belief scale was .93... and .81, .79, .79 and .84 for Scales 1 through 4, respectively" (Carpenter et al., 1988, p. 27). Deemed both valid and reliable, researchers found the MBS an effective tool for measuring teacher beliefs.

In quantitative research, validity and reliability are of utmost concern. According to Creswell (2009), an instrument is valid if it yields data from which a researcher can draw meaningful and useful inferences. An instrument is reliable if it demonstrates internal consistency; item responses are consistent across constructs, scores are stable over time (if the instrument is administered more than once), and administration and scoring procedures are consistent (Creswell, 2009). Developing an instrument that measures what it claims to measure and is both valid and reliable is no easy task. Many instruments take years to develop and improve; the use of an existing instrument deemed both valid and reliable is a practical and logical choice for researchers. Some researchers use an existing instrument as-is while others, as in the case of the MBS, revise an original instrument to create a more efficient, user-friendly version (Capraro, 2001). Sometimes, researchers find it necessary to create an entirely new instrument.

Zollman and Mason (1992) created the Standards Beliefs Instrument (SBI) to assess teachers' beliefs about the NCTM *Standards* (1989) using items representative of the beliefs underlying the *Standards*. Randomly chosen, representative items met three criteria: implications were not intended to be intuitively obvious, items must be clearly stated in a positive or negative manner, and the item's central idea could be incorporated into a single sentence (Zollman & Mason, 1992, p. 359). The sixteen items, based on

themes rather than specific grade-level content, were either near direct quotes or the inverse of direct quotes from the *Standards*. The SBI used a 4-point Likert scale (1 = strongly agree, 2 = agree, 3 = disagree, 4 = strongly disagree) and a demographic questionnaire (Zollman & Mason, 1992). An initial instrument was piloted and revised by teachers with knowledge of the *Standards*; the final version was tested for construct validity by a panel of mathematics education experts (Zollman & Mason, 1992). The SBI was administered to 61 undergraduate practicum teachers and 72 experienced teachers to further assess construct validity—in this case, whether the scores served a useful purpose.

The findings of the study suggested Zollman and Mason's (1992) SBI is a useful instrument to assess teacher beliefs about the NCTM *Standards* (1989) for four reasons. First, the instrument used the same language as the *Standards*. Second, the panel of mathematics education experts determined the instrument represented the *Standards*. Third, convergent and divergent correlations supported the SBI's construct validity. Finally, the reliability of the instrument suggested the survey would produce dependable scores in subjects familiar with the *Standards*.

Zollman and Mason (1992) contended that, in the midst of mathematics education reform (in this case, during the period of *Standards*-based reform efforts), the SBI is a useful tool to encourage “reflective thinking and active decision making regarding... teaching practices” (p. 361). Correlations in the study suggest that mathematics ability, teaching experience, or familiarity with the *Standards* did not necessarily indicate agreement between teacher beliefs and those underlying the *Standards* (Zollman & Mason, 1992).

The findings from this study were relevant to the current period of mathematics education reform, occurring in the wake of widespread CCSSM implementation. The first five of the eight CCSSM Standards for Mathematical Practice (NGA & CCSS, 2010), which describe the ways developing students ought to engage with content, were taken from NCTM *Standards* (1989). While language and underlying vision of the CCSSM and the NCTM *Standards* are similar, two sets of standards are not identical; a new instrument is needed. However, the ability to design a survey that uses the language of academic standards and accurately represents that set of standards—as a whole—is a complex and lengthy process. While it is likely researchers will develop a new instrument to measure teacher beliefs about the CCSSM—in essence an updated version of the SBI—it was beyond the scope of this research project.

There are a variety of instruments besides the MBS and the SBI that use a Likert-style survey to assess certain types of mathematical beliefs, including the Indiana Mathematics Beliefs Scale (Kloosterman & Stage, 1992) which measures student beliefs about mathematical problem solving and the Mathematics Teaching Efficacy Beliefs Instrument (Enochs, Smith, & Huinker, 2000) which assesses pre-service teacher beliefs relating to teacher efficacy. Many researchers elect to use an instrument as-is, others opt to adapt or modify an existing instrument, and others employ a hybrid of instruments to conduct new, but related research. While Likert scale surveys offer researchers an opportunity to collect data from large samples that is convenient for statistical analysis, Likert scale surveys are not the only or, as some researchers claim, best means to measure teacher beliefs.

Alternative Instruments

Adhering to the view that beliefs must be inferred (Pajares, 1992), some researchers feel that inferring an individual's beliefs from closed-item survey responses is less than ideal. In the Integrating Mathematics and Pedagogy (IMAP) project, researchers developed a web-based survey to overcome the three limitations they identified with Likert scale items: they do not provide respondents the opportunity to explain how items were interpreted or their answers, they do not convey the importance of the issue to respondents, and they provide little or no context (Ambrose et al., 2004). The IMAP computer-based survey addresses the limitations of Likert scales by capturing teachers' free response interpretations of scenarios (Ambrose et al., 2004).

Ambrose et al. (2004) cited two beliefs components that account for the significant role beliefs play in teaching and learning and are important to consider when measuring beliefs. First, beliefs influence perception and have a filtering effect on complex situations (Pajares, 1992). Teachers and students constantly experience situations requiring interpretation. The IMAP survey provides respondents with complex situations they are asked to interpret. Second, beliefs as dispositions toward action have a motivational force (Cooney, Shealy, & Arvola, 1998; Rokeach, 1968). When teachers make decisions, their beliefs may compel them to act in particular ways.

To create a survey to assess beliefs that might affect prospective teachers' subsequent learning of mathematics, the IMAP items measured the following seven beliefs, organized into 3 categories:

Belief About Mathematics

- Belief 1. Mathematics, including school mathematics, is a web of interrelated concepts and procedures.

Beliefs About Knowing/Learning Mathematics

- Belief 2. One's knowledge of how to apply mathematical procedures does not necessarily go with understanding of the underlying concepts.
- Belief 3. Understanding mathematical concepts is more powerful and more generative than remembering mathematical procedures.
- Belief 4. If students learn mathematical concepts before they learn procedures, they are more likely to understand the procedures when they learn them. If they learn the procedures first, they are less likely ever to learn the concepts.

Beliefs About Children's [Students'] Doing and Learning Mathematics

- Belief 5. Children can solve problems in novel ways before being taught how to solve such problems. Children in primary grades generally understand more mathematics and have more flexible solution strategies than their teachers, or even their parents, expect.
- Belief 6. The ways children think about mathematics are generally different from the ways adults would expect them to think about mathematics. For example, real-world contexts support children's initial thinking whereas symbols do not.

Belief 7. During interactions related to the learning of mathematics, the teacher should allow the children to do as much of the thinking as possible. (Ambrose, et al., 2004, p. 4)

The survey captured qualitative data that later quantified, for purposes of comparison, using a systematic method for creating research rubrics. Provided with video-taped scenarios, respondents were asked their reactions and to identify teaching strengths and weaknesses. These open responses were placed on a rubric (no evidence, weak evidence, evidence, strong evidence) that the teacher holds one of the seven beliefs about teaching and learning mathematics assessed in the study. The IMAP survey, collecting both quantitative data to measure change in individuals and for comparing individuals to one another and qualitative data that offered deeper insight into respondents' beliefs and interpretations, provided a more valid measure than a Likert-style instrument. Pre/post administration survey results indicated the IMAP was an effective tool for assessing belief change (Ambrose et al., 2004). Instruments, like the IMAP survey, that employ an alternative design address some of the limitations of Likert scale items—particularly important when measuring a complex construct such as beliefs. IMAP researchers concluded that one strength of their instrument is the ability to provide context to gather multi-purpose qualitative data, but also acknowledged that “this strength comes with a cost in terms of time required” (Ambrose et al., 2003, p. 8). Developing and analyzing the results of such an instrument requires vast resources; again, these resources were beyond the scope of this study.

Synthesis of Methodological Literature on Teacher Beliefs and Practice

Philipp (2007) identifies two common approaches in studies on teacher beliefs in mathematics education, each presenting both strengths and limitations. The case-study approach provides rich, qualitative data (often triangulated) from a variety of sources including classroom observations, interviews, responses to vignettes, etc. (Philipp, 2007, pp. 268, 271). This approach, while useful for building theory, is often too expensive to administer to a large number of participants. Another approach, using Likert scale surveys, can test theory by generating large data sets. These instruments, though valid and reliable, limit respondents' abilities to explain response or item interpretations and may not provide context—important when inferring beliefs. Alternative means to assess beliefs include open-response items that generate qualitative data that can later be quantified for comparison. Both Likert and non-Likert scale surveys require considerable resources to develop.

Conclusion

The study of beliefs, a “messy construct” (Pajares, 1992), requires researchers to clearly define the construct, narrow the focus by examining particular beliefs, and choose appropriate means to study them (Pajares, 1992; Philipp, 2007). The purpose of this study was to answer two research questions. First, what do teachers believe about mathematics, teaching mathematics, and learning mathematics? Second, how do these beliefs influence their interpretation and implementation of the Common Core State Standards for Mathematics?

For this study, beliefs are defined as “psychologically held understandings, premises, or propositions about the world that are thought to be true” (Philipp, 2007, p. 259). The theoretical perspective that guided this research views beliefs as the best indicators of decisions and actions (Bandura, 1986; Dewey, 1910). As stated in the first research question, this investigation focused specifically on the beliefs teachers hold about mathematics as discipline, mathematics pedagogy, and how students learn mathematics. Extensive research has been conducted on the teacher beliefs-practice relationship. Beliefs are powerful filters through which new phenomena are interpreted (Pajares, 1992). Therefore, the researcher hypothesized that teachers who hold very different beliefs (traditional or low-constructivist versus high constructivist) would interpret the new phenomenon of the CCSSM in different ways which would, in turn, yield different ways of implementing the CCSSM in the classroom.

Beliefs, as complex constructs, are difficult to study. Not only must researchers clearly define the construct and narrow the focus, they must also determine the most appropriate methodologies to answer the research question. A review of the research on teacher beliefs related to practice indicated two main approaches: the case-study approach and the beliefs-assessment instrument (Philipp, 2007). Each approach offers strengths as well as limitations. To capitalize on the strengths and mitigate the limitations of using only one approach, this research study used a mixed methods design, employing both a mathematical beliefs instrument for a larger population of teachers as well as interviews with certain teachers selected from that larger population. Rich data sets, gathered using both quantitative and qualitative methods, yielded a more comprehensive

understanding of teachers' mathematical beliefs and how those beliefs influence their interpretation and implementation of the CCSSM.

Using a beliefs-assessment instrument offers researchers the ability to gather a large data set. To answer the first question regarding what teachers believe (about mathematics, teaching mathematics, and learning mathematics), the use of an existing beliefs-assessment instrument was a pragmatic choice. The Mathematics Beliefs Scale (MBS), developed and tested by Fennema et al. (1990), was found to be both valid and reliable by the original authors as well as other mathematics education researchers (Capraro, 2001; Philipp, 2007). Creating an entirely new instrument, or even employing a modified or hybrid version of an existing survey, was not practical given the resources available for this study. The MBS, therefore, was chosen as an effective, appropriate instrument to measure teachers' mathematical beliefs.

Using the quantitative data set, according to methods described by Fennema et al. (1990), a smaller group of teachers was identified for one-on-one, explanatory interviews—generating qualitative data to further explain the statistical findings of the survey and to address the limitations of the survey instrument. Thus, the mixed methods design, using both survey and interview, offered a pragmatic approach appropriate for answering both research questions and aligning with the theoretical and methodological framework presented in this review, as well with the researcher's philosophical foundations.

A bricoleur constructs using any tools available. Acting as a bricoleur, the mathematics education researcher appreciates the complexities of the teacher beliefs-

practice relationship; the use of one philosophical lens (or tool) may lead to a limited understanding of such a complex construct. Cobb (2007) suggested it is not necessary for researchers to choose one perspective; they may adapt “ideas from a range of theoretical sources” (p. 29). Kvale and Brinkmann (2009) extend the bricolage research approach to the analytical phase of research as well.

The mixed methods design afforded the structure of an existing, effective survey instrument to gather quantitative data to answer the *what* of the first research question complemented by the flexibility of personal interviews from which to gather qualitative data to answer the *how* contained in the second research question. To date, no studies have examined the relationship between teachers’ mathematical beliefs and practices related to the CCSSM. Lack of research indicated the need for further inquiry and this study’s mixed method design contributed to knowledge in mathematics education by building on the findings and progress of other researchers in the field while also addressing the limitations of these earlier efforts.

Based on the theory of the powerful effect of beliefs, it follows that teacher beliefs about mathematics, teaching mathematics, and learning mathematics will act as a filter through which they interpret the CCSSM and, ultimately, implement these new mathematics standards in the classroom. Contributing to a better understanding of beliefs, perhaps “the single most important construct in educational research” (Fenstermacher, 1979 as cited in Pajares, 1992, p. 329), is a worthwhile research endeavor. The methodology used to pursue this endeavor is presented in the following chapter.

CHAPTER THREE: METHODS

Introduction

Although mathematics education as an activity goes back centuries, it has only been since the end of the 19th century that mathematics education has emerged as a professional field (Kilpatrick, 1992). Research in mathematics education, defined as the “disciplined inquiry into the teaching and learning of mathematics,” has three main purposes: (a) to explain, predict, or control—in the empirical-analytical tradition; (b) to understand meanings that learning and teaching of mathematics have for those engaged in the activity—in the cultural-anthropological tradition; and (c) to conduct “action-research” to give teachers and students a voice in the field—in the critical-sociological tradition (Kilpatrick, 1992, pp. 3-4). One’s purpose for research and perspective of what constitutes research play a significant role in deciding which questions are asked and which methods are used to answer them. This chapter reviews the research purpose for this study and describes the overall research design, methods, procedures, and instruments used for data collection, organization, and analysis. The researcher will also present expected findings related to the questions posed in the study.

Purpose of the Study

Considering the powerful effect beliefs have on teachers’ practices, it is important to examine how beliefs may impact the decisions and actions teachers make when enacting educational reform, policy, and standards. The intent of this two-phase, sequential mixed methods study was to better understand the relationship between teachers’ mathematical beliefs and their practices relating to the Common Core State

Standards for Mathematics (CCSSM). The CCSSM present a gap in, as well as an opportunity to, research teachers' mathematics beliefs in relation to a new phenomenon to contribute to knowledge in the mathematics education field. Specifically, this study investigated:

- 1) *What do teachers believe about mathematics, teaching mathematics, and learning mathematics?*
- 2) *How do these beliefs influence their interpretation and implementation of the Common Core State Standards for Mathematics?*

Each of these two research questions required a different approach of inquiry. The explanatory mixed methods design (Creswell, 2009) produced a rich set containing both quantitative data—to answer the *what* of the first research question—as well as qualitative data—to answer the *how* of the second question.

Research Design

There is a relationship between the character of the research community and the methods it employs. Choice of methods indicates the perspectives and values of the community conducting the research (Schoenfeld, 1994). Both mathematics and psychology have shaped mathematics education research. Over time, an early emphasis on quantitative studies reflecting the perspective of the mathematics field, which valued scientific (objective) inquiry, shifted to include qualitative studies that used methods similar to those introduced in the cognitive sciences (Schoenfeld, 1994). This progression toward a more eclectic approach to research, precisely the philosophical and

methodological bricolage recommended by Cobb (2007), is useful not only across the mathematics education research field, but also within the scope of a single study.

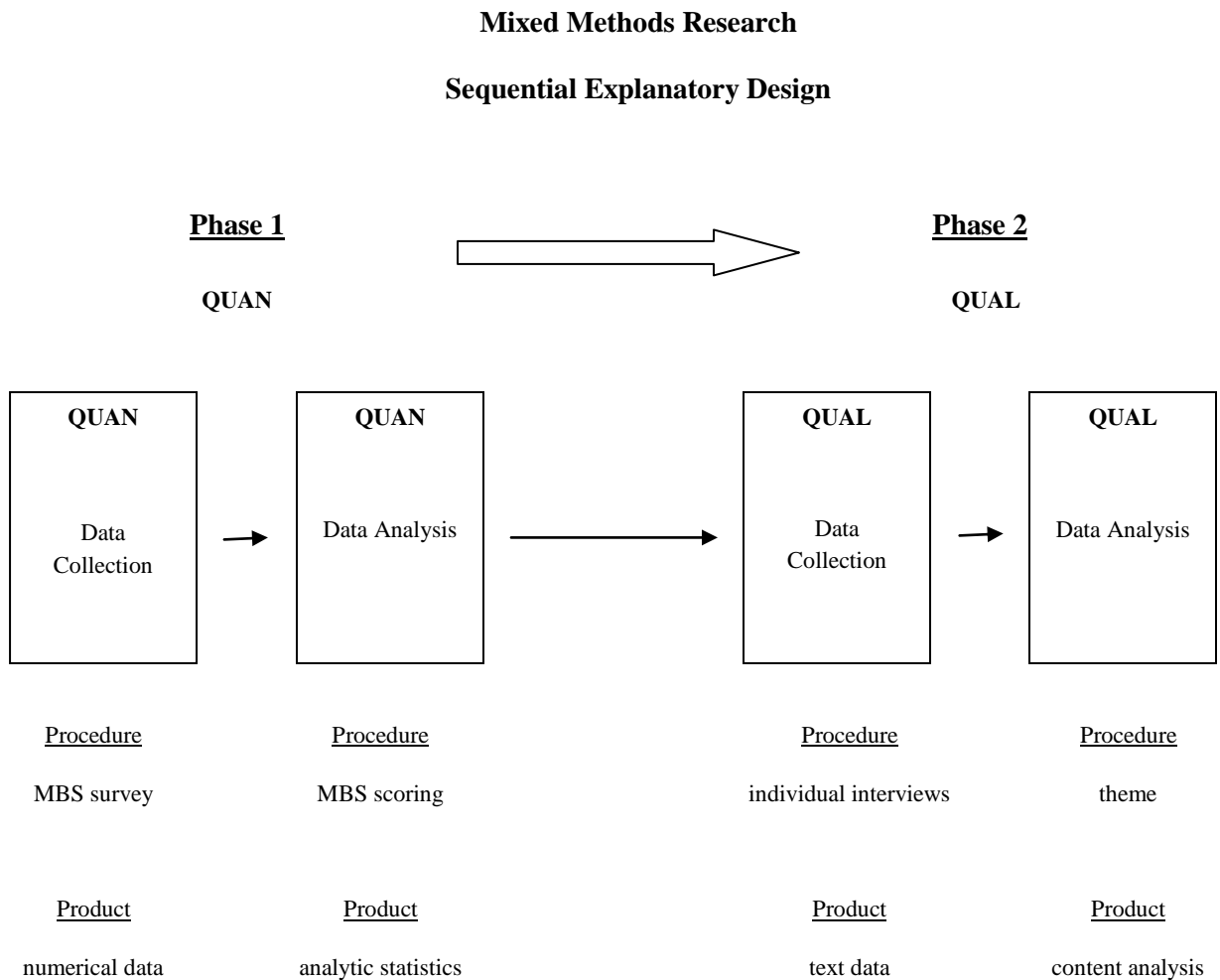
Mixed Methods Research

Campbell and Fiske (1959) introduced the mixing of research methods, using multiple approaches to test validity in the psychology field. Mixed methods researchers feel that “biases inherent in any single method could neutralize or cancel the biases of other methods (Creswell, 2009, p. 14). For this study, the sequence in which the research questions are asked required an explanatory design (see Figure 1), using “qualitative questions that provide explanations for findings from quantitative questions” (McMillan & Schumacher, 2010).

In the first research phase, the researcher gathered quantitative data from a Likert-style survey designed to assess mathematical beliefs. This data informed the selection of participants for the second research phase in which the researcher gathered qualitative data from interviews focused on the ways each individual teacher interprets and implements specific grade-level standards from the CCSSM. Thus, as with many explanatory designs, this study “begins with a broad survey... and then focuses, in a second phase, on detailed qualitative, open- ended interviews to collect detailed views from participants” (Creswell, 2009, p. 18).

This two-phase research approach—first asking questions to generate statistical findings about *what* teachers believe, followed by qualitative questions to determine *how* those beliefs influence their interpretation and implementation of specific mathematics standards—suited the practical purpose for this research, the research questions

Figure 1

Research Design Diagram

investigated in the study, and the complex relationship between beliefs and practice.

Unlike researchers who choose to use *either* a quantitative *or* a qualitative method, the mixed methods researcher adopts a *both-and* pragmatic world-view and research approach.

Pragmatists adopt an integrated worldview, accepting the existence of *both* the internal world (within the mind) *and* the external world, to focus on problems at-hand and use a variety of tools to solve it (Creswell, 2009). This approach parallels the research problem posed in the study: the CCSSM exist as an external document of educational reform, policy, and standards, which is then internalized by each individual teacher through the filter of his or her mathematical beliefs. The pragmatic, mixed methods approach “opens the door to multiple methods, different worldviews, and different assumptions, as well as to different forms of data collection and analysis” (Creswell, 2009, p. 11). For this study, the explanatory (mixed method) research design offered complementary approaches to examine the complex nature of teacher beliefs and their relationship to professional practice—both the structure of predetermined, ordered survey questions and the flexibility of an interview in which participants had the freedom to communicate personal experiences of interpreting and implementing specific CCSSM standards. This eclectic research model suited the complex nature of beliefs, addressed the challenges associated with studying them, and strengthened the overall design by mitigating the limitations inherent when using only one research approach.

Survey and Interview Methods

As with any research method, the survey and interview each offers both strengths and weaknesses. The advantages of using a survey include the “economy of the design and the rapid turnaround in data collection” (Creswell, 2009, p. 146). The survey method enables researchers to efficiently gather large amounts of numerical data and, in this

study, will focus the types of beliefs to be assessed (Pajares, 1992; Philipp, 2007). While the survey method is an advantageous research tool, it is not without flaws.

Likert-style survey instruments, the type used in this study, offer an efficient means of gathering numerical information, but lack the context usually required to capture sensitive data about complex phenomena—like teachers' mathematical beliefs (Ambrose et al., 2003; Pajares, 1992). Participants may not interpret the statements in the way intended by the survey's original author(s) or this researcher. Survey instruments can be costly and time consuming to develop and test for standards for reliability and validity (Ambrose et al., 2003). Finally, surveys rely on participants to respond thoughtfully and truthfully. While these issues should not be underestimated, there are ways to address them.

To reduce the time and cost of creating and testing a new instrument, the researcher can instead use an existing survey proven to be reliable and valid. To encourage accurate self-reporting of mathematical beliefs, assigning identification numbers to participants ensured confidentiality of responses. Protecting participants in this way likely increased their willingness to respond truthfully in the sense that responses would not be made public—either among colleagues or district administrators. Using a valid and reliable instrument and protecting participants enabled the researcher to gather information useful in addressing the first research question in this study.

The second research question in this study asked *how* teachers' mathematical beliefs influence their practices relating to the CCSSM. Consistent with the view that beliefs are the most powerful indicators for decision and actions of individuals (Dewey,

1910), including classroom teachers (Pajares, 1992), is the recognition that to probe this beliefs-practice relationship beyond the surface required an additional research method. The interview allowed the researcher to listen to teachers explain the meaning they make and the ways they teach specific standards, resulting in a richer data set to complement and further explain the quantitative data. The interview method addressed limitations inherent in the Likert-type survey method: lack of context and clarification.

Personal interviews provided the context needed for generating qualitative data from which to infer teacher beliefs and their influences on practice. The semi-structured interview enabled participants to explain in their own language how they interpret specific grade level mathematics standards from the CCSSM as well as how they implement the standards in the classroom. The researcher asked follow-up questions to clarify participant views. However, the interview method presents its own limitations.

Interviews are time intensive and only reflect the views of a small number of participants, which may not be useful to build theory (Philipp, 2007). Each part of the process—conducting, transcribing, and analyzing the interview—takes a considerable amount of time to complete. For this study, it was not reasonable to conduct individual interviews with all teacher participants. However, by combining the survey and interview methods into a single study, the strengths of each method was used to address the weaknesses of the other.

In mixed methods research, Creswell (2009, pp. 208-209) identifies four important aspects to consider:

1. timing
2. weighting
3. mixing
4. theorizing

The research questions in this study (asking both *what* and *how*) suggested an explanatory mixed methods design in which quantitative methods in the first phase of research informed the qualitative methods in the second phase with equal weighting assigned to each phase. The purpose of the research was to better understand the relationship between beliefs and practice—both were essential components of the overall research aim. The third aspect, mixing, refers to when and how the two kinds of data are blended. For this study, the two data bases were connected and combined during the two phases of research. Connected, in this sense, means the “mixing of the quantitative and qualitative research are connected between a data analysis of the first phase of research and the data collection of the second phase of research” (Creswell, 2009, p. 208). The analysis of the quantitative data gathered during the first phase identified participants for the follow-up qualitative phase. During the second phase, the quantitative data from the survey (first phase) was combined with the qualitative text data from the interview to provide a more comprehensive understanding of each participant’s mathematical beliefs and practices. Finally, the theoretical perspective guiding this study was that beliefs are the best indicators of decisions and actions (Dewey, 1910). Therefore, teachers who hold very different beliefs—about what mathematics is, how mathematics should be taught,

and how students learn mathematics—may interpret and implement a common mathematics standard from the CCSSM differently.

Research Site

The research site was a regional public school district serving approximately 3500 students in kindergarten through eighth grade in the Mid-Atlantic region of the United States. Five of the six schools in the district were involved in the study: four elementary schools (K-4) and one intermediate (5-6) school. Data was collected in February through April, 2015. The school district does not offer open enrollment; only students living within the district’s geographic boundaries are eligible to attend its schools. Table 1 summarizes the comparative demographic information for schools involved in the study.

Table 1

School Demographics: 2013-2014 School Year

	School 1	School 2	School 3	School 4	School 5
American Indian	0.3%	0.0%	0.0%	0.3%	0.0%
Asian	6.4%	5.5%	4.6%	9.9%	6.6%
Black	1.6%	3.2%	2.9%	3.8%	3.8%
Hispanic	4.3%	9.3%	27.0%	15.8%	7.4%
Pacific Islander	1.1%	0.0%	0.2%	0.0%	0.1%
Two or More Races	4.5%	1.5%	1.5%	0.5%	0.3%
White	81.9%	80.4%	63.7%	69.7%	81.8%
Limited English Proficiency	0.0%	4.2%	15.0%	5.3%	0.4%
Students with Disability	18.0%	20%	8.0%	15%	16%
Economically Disadvantaged	4.0%	7.4%	30.1%	21.1%	9.9%
Total Enrollment	379	526	455	393	792

The research site was chosen by the researcher for two reasons. First, the site was geographically convenient to the researcher, who is also a parent of children enrolled in the district. Parental contact with the teaching and administrative staff had been limited to four volunteering experiences and parent-teacher conferences in the school year prior to and during this study. All other contact was directly related to this study, which was initiated in the spring of 2014 and began formally in November, 2014. Second, at the time the research was conducted, the district was in the middle of its fourth year implementing the two-part CCSS (adopted in 2010), including the CCSSM. The timing of this study was both ideal and tenuous. Three and one-half years of experience with the CCSSM, and the district's professional development and training sessions relating to the CCSSM, made teachers more comfortable with these new mathematics standards. This level of familiarity with the CCSSM likely increased the ability of teachers to discuss them knowledgeably. The timing of this study also, however, presented a unique challenge.

For the first time since CCSSM adoption, the district was implementing accountability measures related to the standards by evaluating teacher performance based, in part, on student test scores on the 2015 spring Partnership for Assessment of Readiness for College and Careers (PARCC). PARCC developed a set of assessments, aligned with the CCSS to "ensure that every child is on a path to college and career readiness by measuring what students should know at each grade level" (PARCC, 2014, About PARCC section, paras. 1-2). The PARCC was designed "based on the core belief that assessment should work as a tool for enhancing teaching and learning" that will

“provide parents and teachers with timely information to identify students who may be falling behind and need extra help” (PARCC, 2014, About PARCC section, para. 2).

However, the use of test scores to evaluate teachers as a measure of accountability created an additional factor for teachers and administrators to consider that was not in place during the first three years of CCSSM implementation. The increased need for educational research related to the CCSSM coming at the same time external pressures for accountability were put in place presents a paradox similar to what occurred in the mathematics education field a decade earlier.

The No Child Left Behind Act (2002) had paradoxical implications for mathematics education research in schools: the importance of research was elevated and there was greater support and funding for such research, but the increased pressure for schools to perform made school-based research partnership challenging to find (Chval, Reys, Reys, Tarr, & Chavez, 2006). The seemingly ideal climate for scientifically-based research was actually hampered by stronger accountability for schools, more data to collect, analyze, and report, and public access to school performance information. To encourage more practitioner participation in the research process, Chval et al. (2006) suggested researchers can overcome politically contentious education climates by emphasizing the common goal of researchers and practitioners—to improve the teaching and learning of mathematics. For this study, in a similar climate more than a decade later, this researcher and the district’s administrators and teachers recognized the benefit of researching mathematical beliefs and the different ways in which teachers interpret and implement the CCSSM in individual classrooms. Understanding diverse views and

practices associated with the standards may improve student learning in the district. The district and its teachers were, therefore, willing participants in the research process and collaborative partners in this specific study.

Participants

Participant selection was driven by the objective of the research, which is to better understand how the mathematical beliefs teachers hold shape their practices relating to the CCSSM. While the entire district population of 300 licensed teachers hold educational beliefs and are—directly or indirectly—responsible for interpreting and implementing academic standards (including the two-part CCSS), this study focused exclusively on mathematical beliefs relating to the CCSSM. Therefore, the participant sample for the first phase of the study consisted of 80 licensed teachers in the district who were directly responsible for teaching mathematics to students at one of the four elementary (K-4) schools or the intermediate school (5-6) as part of their official teaching responsibilities in any educational setting, including the traditional classroom as well as special education, intervention, learning support, or enrichment programs. Years of teaching experience or additional licensure or professional development in mathematics were collected as part of the questionnaire, but were not considered inclusion or exclusion criteria. Thus, there are two inclusion variables in the sample selection process: grade level (K-6) and mathematics teaching as an official professional responsibility. Other licensed teachers in the district who do not meet these criteria were excluded from the study.

Limiting the teacher population and sample of participants to a single school district was appropriate for the resources allocated for this research project. It also reduced the number of non-beliefs variables that, as indicated in the literature, also influence interpretation and implementation of educational reform, policy, and standards. Some of these variables include teacher knowledge, curriculum, and district teacher training, support, professional development, and the student population served.

The 80 licensed teachers who fit these criteria were included in the study and asked to complete a demographic questionnaire and survey called the Mathematics Beliefs Scales (MBS) (Fennema et al., 1990). The MBS instrument has been used in several studies since its creation in 1990, ranging in sample size depending on the specific research questions and design. For example, the original developers (Fennema et al., 1996) used the MBS instrument to measure changes in the mathematical beliefs of a small sample ($n=21$) of practicing teachers. Three years later, another study that used the MBS to measure changes in the mathematical beliefs of prospective teachers (Vacc & Bright, 1999), involved a sample size of 34 teachers enrolled in a teacher education program. Researchers in another study, attempting to design a more efficient version of the MBS, administered the original survey to samples of both practicing ($n=123$) and preservice ($n=54$) teachers (Capraro, 2001). In this study, the purpose of measuring mathematics beliefs in the first research phase was to inform the second research phase. This sample size ($n=80$) generated sufficient quantitative data to reflect differences in the mathematical beliefs of teachers at each grade level (K-6) and identify potential

participants for the qualitative phase of the study and was reasonable considering the resources available for this study.

Participant Protection

In accordance with school district policy and the human subjects committee at the researcher's university, steps were taken to ensure the protections of teacher participants. The informed consent letter (Appendix D) described the study's research purpose, instruments, any risks associated with participation, and the protocol to ensure confidentiality of the data. To protect the identities of the participants, the researcher placed three-digit identification numbers on each questionnaire and survey packet. Upon returning completed packets, each participant recorded his or her name and identification number on a meeting attendance sheet that was kept in a secure location by the district or school representative. The researcher had no access to the list of participant names and identification numbers at any time during the study. Thus, during the first phase, participant survey and questionnaire responses were anonymous. Only the identities of participants selected for interviews during the second, qualitative research phase were given to the researcher. Interview participant identities were kept strictly confidential. No names or other identifying information about individual teachers, schools, or the district were included in the raw data set, the dissertation submitted to the researcher's university, or in any future material created about the study. Only general study findings were reported.

Phase I: Quantitative Methods and Procedures

During the months of October, November, and December, 2014 the researcher met and corresponded via electronic mail with one elementary school administrator and the district science and mathematics supervisor, who agreed to serve as district representatives for this project. Together, these individuals and the researcher determined the study protocol, including the logistics for administering the survey, securing confidentiality, and conducting interviews. In January, 2015, the researcher met with the school district's assistant superintendent to discuss the project. The assistant superintendent agreed to propose the study to the school board during a regularly scheduled meeting and granted a letter of conditional approval for the project. The school board granted final approval for the study later in January, 2015. The demographic questionnaire and survey was scheduled for administration during professional development meeting time in February or March, 2015.

Quantitative Instruments

Demographic questionnaire. The demographic questionnaire (Appendix B) is a two-page instrument, designed by the researcher, comprised of both open and closed items designed to capture information regarding professional teaching and development and/or educational experiences: current and previous teaching assignments—grade level(s) and setting(s); years of teaching experience—within and outside of the district; teaching licensure(s); educational background, and additional professional development and training relating specifically to mathematics. The demographic questionnaire was an appropriate instrument that generated data in the first phase of research—the grade level

assignment for each participant—and additional information to select participants for interviews conducted during the second research phase.

Survey. Engaging a post-positivist lens, the survey instrument attempts to “measure the objective reality that exists ‘out there’” (Creswell, 2009, p. 7). For this study, ‘out there’ was the public school classroom. The survey can “describe, compare, or explain individual and societal knowledge, feelings, values, preferences, and behavior” (Fink, 2009, p. 1) and is both versatile and efficient (McMillan & Schumacher, 2010). Creswell (2009) confirms that “standards of validity and reliability are important in quantitative research” (p. 7).

To ensure these standards of validity and reliability are met, the researcher used an existing teacher beliefs survey, specifically designed to measure what teachers believe about mathematics, teaching mathematics, and learning mathematics, which has been tested and used successfully by other mathematics education researchers. Response item analysis indicated correlational relationships among beliefs and tested the theory that not all teachers hold identical beliefs.

Mathematics Beliefs Scales (Fennema, Carpenter, & Loef, 1990). Originally developed and tested by Fennema et al. (1990), the MBS is a paper-and-pencil Likert-type instrument of 48 statements designed to assess teachers’ mathematical beliefs in four areas, or subscales:

- Role of the Learner—how children learn mathematics
- Relationship Between Skills and Understanding—whether skills should be taught in relation to conceptual understanding and problem solving

- Sequencing of Topics—the order in which mathematical topics should be taught and learned
- Role of the Teacher—how mathematics should be taught

For each item, the participant chooses a letter to indicate his or her level of (dis)agreement with the statement: A =Strongly Agree, B=Agree, C =Undecided, D=Disagree, E= Strongly Disagree. Beliefs associated with each of the four scales are assessed using statements written in language suitable for teachers of both elementary and middle school students.

Fennema et al. (1990) created the MBS by adapting an earlier version of the scales (Fennema, et al., 1987) to measure changes in teacher mathematical beliefs after participating in the Cognitively Guided Instruction program. The MBS specifically has been shown to be a valid, reliable, and effective instrument by the original developers (Fennema et al., 1990) as well as other researchers (such as Vacc & Bright, 1999 and Capraro, 2001) over several decades.

Using Cronbach's alpha, on a sample of 39 teachers, the "internal consistency of teachers' scores on the total belief scale was .93... and .81, .79, .79 and .84 for Scales 1 through 4, respectively" (Carpenter et al., 1988, p. 27). Other researchers using the MBS have found the coefficient-alpha reliability of scores is also acceptable, including the .78 reliability of scores found by Capraro (2001). While most of the researchers using the MBS focused on changes in teacher beliefs, the survey is also a valid, reliable, and useful tool when administered as a survey to assess teachers' mathematical beliefs at a given point in time. As a proven quantitative instrument, used in several studies over more than

two decades, using the MBS as-is did not require resources to create or pilot test an entirely new survey.

Quantitative Data Collection

The data collection procedures for the first (quantitative) research phase included an informal introduction to the study from each school administrator to his or her teaching staff in February, 2015, to make teachers aware of the upcoming study and communicate school board approval for the research project. In February and March, 2015, the researcher or district mathematics and science supervisor met with entire grade levels of teachers at the district or school level during professional development meeting time to reintroduce the study and distribute the informed consent letters (see Appendix D), demographic questionnaires (see Appendix A), and survey (see Appendix B). Both the researcher and the district mathematics and science supervisor read from a script to keep study information and language consistent. Only teachers eligible for participation in the study attended these meetings.

The researcher introduced and administered the questionnaire and survey to all district first and second grade teachers on the morning of Friday, February 13, 2015 as part of a district level professional development meeting. In the afternoon, teachers returned to their individual buildings for school-level sessions. On the afternoon of February 13, 2015, the researcher met with third and fourth grade teachers at two of the four elementary schools while the district mathematics and science supervisor met with district kindergarten teachers at another district school to introduce and administer the questionnaire and survey. Two of the four elementary school principals elected not to

have their third and fourth grade teachers participate in the study, citing upcoming PARCC standardized tests, mid-year evaluations, and low morale as primary factors in their decisions. These decisions reflect the effect of accountability measures (such as the use of standardized tests to evaluate district, school, and teacher performance) on educational research found by Chval et al. (2006) nearly a decade earlier. The researcher collected signed informed consent letters and questionnaire and survey materials that afternoon for teachers in grades one, two, three, and four and from kindergarten teachers the following Monday, February 16, 2015 from the secure office of the district mathematics and science supervisor.

Two teachers at one elementary who were unable to attend the afternoon session at their school (due to attending another district meeting scheduled at the same time) wished to participate in the study. Their school principal, who also served as the district representative in the development phase of the study and worked closely with the researcher and district mathematics and science supervisor, introduced the study and distributed materials which were returned to the researcher within a one-week period.

Due to weather-related school cancellations in late February and early March, 2015, the professional development schedule for the district's fifth and sixth grade teachers was significantly altered. Therefore, the math and science supervisor met with those teachers on Thursday, March 13, 2015 to introduce and distribute questionnaire and survey materials with a request to complete and return materials to the supervisor's secure office (located in a building connected to the intermediate school) on Monday, March 16, 2015.

Participating teachers completed three forms: the informed consent, the questionnaire, and the survey. Unless otherwise arranged, at the end of the meeting time, the supervisor or researcher collected the informed consent letters, demographic questionnaires, and surveys from the participants; all materials were stored in a locked drawer in each school office until collected by the researcher. District representatives and the researcher wanted to ensure a high response rate by providing time during a regularly scheduled staff meeting. The thirty-minute period allowed adequate time for teachers to read, consider, and respond thoughtfully.

Questionnaire and survey packet identification numbers were recorded on meeting attendance sheets. Informed consent letters were signed and returned by all teacher participants. Once the researcher collected study materials (not including the meeting attendance sheets), all items were secured in a locked home office. As indicated in the informed consent letter, all study materials will be destroyed after the successful defense of the dissertation, expected no later than September 1, 2015.

To organize the quantitative data for analysis, the researcher entered all participant responses from the paper-and-pen demographic questionnaire and MBS survey into a Microsoft 2007 Excel spreadsheet, tracking data by identification number and sorted by grade level.

Quantitative Data Organization and Analysis

This study used descriptive statistics, “techniques used to summarize a set of numbers” to “see information more clearly” (Reid, 2013, p. 4). Specifically, the use of summary statistics—or single values—for the quantitative data were used to infer

the degree (low to high) to which a participant's mathematical beliefs are constructivist, which informed the selection of participants used to gather qualitative data. Measures of variability showed the dispersion of participant scores which the researcher used for categorization and comparison.

For each item on the MBS survey, the participants chose a letter to indicate his or her level of (dis)agreement with the statement: A =Strongly Agree, B=Agree, C =Undecided, D= Disagree, E= Strongly Disagree. When entering participant data (using district-provided identification numbers to protect identity) into the Microsoft 2007 Excel spreadsheet, the researcher coded response items according to the procedure used by the MBS developers. For positive statements, A=5, B=4, C=3, D=2, E=1. Negative statements (items 5, 7, 8, 11, 14, 15, 16, 17, 18, 22, 23, 24, 26, 29, 34, 35, 38, 39, 42, 44, 45, 46, 47, and 48) were coded in the opposite direction: A=1, B=2, C= 3, D=4, E=5. The researcher added each participant's numerical values (for all responses) to determine the sum score which was then divided by 48 (total number of items) to generate an overall mean score.

The researcher used a summative scale to align participants according to overall mean score. Each teacher's mean score reflected mathematical beliefs consistent with very low-constructivist (VLC), low-constructivist (LC), constructivist I, or high-constructivist (HC) views of teaching and learning mathematics. Overall mean scores were placed into one of four categories:

Overall Mean Score	Categorization
1 – 1.99	very low-constructivist (VLC)
2 - 2.99	low-constructivist (LC)
3 - 3.99	constructivist (C)
4 – 4.99	high-constructivist (HC)

Constructivism. Just as the term beliefs can be interpreted in different ways, the word constructivist holds a variety of meanings in the field of education. Roots of learning as construction can be traced to the eighteenth century Italian philosopher Giambattista Vico (as cited in Cobb, 2007) who proposed that “the known is made” (p. 20). This perspective influenced leaders in the fields of education and cognitive science to rethink and reject the traditional education model with a view of the learner as a passive recipient of knowledge, an empty vessel waiting to be filled with knowledge by the teacher. These leaders (including Vygotsky, Piaget, and Montessori among others) advocated a constructivist education model which described learning as a “self-regulatory process of struggling with the conflict between existing personal models of the world and discrepant new insights” (Fosnot, 2005, p. ix). Constructivist theory views learning as an active process in which each individual builds on prior knowledge and creates representations of the world based on his or her unique perspective.

Items on the MBS (Fennema et al., 1990) are written as statements reflecting either constructivist (positively scored) or non-constructivist (negatively scored) beliefs about mathematics education. Higher scores on Role of the Learner items indicate a

belief that children construct their own mathematical knowledge, while lower scores suggest the belief that children receive knowledge from their teacher. For items focused on the Relationship Between Skills and Understanding, a higher score reflects the belief that students can solve real-world problems while learning computation skills whereas a lower score aligns with the belief that computation skills are prerequisite for solving word problems. Role of the Teacher items assess how teachers should teach mathematics. A higher score indicates the belief that teachers facilitate student learning; a lower score suggests the belief that teachers should show and tell students how to solve problems. Finally, higher scores on items related to the Sequence of Topics suggest using children's natural development in mathematical thinking to guide instruction; lower scores on these items reflect the beliefs that topics should be presented based primarily on the formal structure of mathematics.

Capraro (2001) conducted a study to determine whether a more efficient version of the MBS could effectively measure teacher mathematics beliefs. Capraro performed a factor analysis by investigating data patterns. Findings suggested a reduction in survey length to 18 items using a three-factor subscale (2001). The three subscales included Stages of Learning (combining the Relationship Between Skills and Understanding and the Sequencing of Topics subscales), Student Learning, and Teacher Practices:

Stages of Learning	beliefs about the sequencing of topics and whether computational skills should precede solving word problems
Student Learning	beliefs about whether students can construct their own knowledge or receive most knowledge from a teacher

Teacher Practices beliefs about whether teachers should facilitate student learning or direct student learning (Capraro, 2001, pp. 12-13)

For the purposes of this study, the original 48-item MBS, as published in Capraro (2001) was used along with a three-factor belief framework shaped by revision study and related studies such as Ambrose, et al. (2004). The three belief factors in this study included:

- Mathematics—beliefs about the relationship among skills (basic facts and computation procedures), conceptual understanding, and word problem solving; the sequencing of mathematical topics
- Learning Mathematics—beliefs about how students learn mathematics
- Teaching Mathematics—beliefs about how mathematics should be taught

The sum of raw item scores and an overall mean score indicated the degree of constructivist beliefs held by each teacher participant. Individual survey item responses were used in combination with interview responses to better understand the connection between what teachers believe and how they act on those beliefs in the classroom in relation to each of the three belief factors.

In the sequential explanatory research design, the researcher organized and analyzed the quantitative data to identify potential participants for the qualitative research phase. For this study, the raw data collected from the demographic questionnaire and the scores from the MBS were organized into separate Microsoft 2007 Excel spreadsheets according to the grade level data indicated in the demographic questionnaire. The

researcher used a distribution of overall mean scores to place each participant along a continuum from very low- to high- constructivist mathematical beliefs. To investigate how beliefs influence the ways teachers interpret and implement mathematics standards, the researcher used numeric data to identify same-grade teachers who hold different mathematical beliefs—low- or high-constructivist—as assessed by the MBS. Thus, for each grade level, participants with the greatest range in mean scores were targeted to participate in the second research phase.

Quantitative Research Question and Hypothesis

This sequential explanatory study design addressed two related research questions. In the quantitative phase of research, data was collected to answer the first research question:

- 1) *What do teachers believe about mathematics, teaching mathematics, and learning mathematics?*

Based on the theoretical literature, including the empirical findings of other researchers who have examined teachers' mathematical beliefs, the researcher expected to find differences in the mathematical beliefs teachers hold. Given the number of sources (philosophical, cultural, lived experience) that influence beliefs, it is a reasonable conjecture that, even with a single school district, there would be differences in the mathematics beliefs teachers hold.

The researcher conjectured that some teachers in the sample, as determined by their mean scores on the MBS instrument, would hold low-constructivist beliefs about mathematics, teaching mathematics, and learning mathematics while other teachers

would hold high-constructivist beliefs. To test these expectations, the researcher used descriptive statistics to:

- 1) describe survey participants
- 2) describe their responses
- 3) generate a numeric value from which to determine differences in the mathematics beliefs of the participants in the sample and, if found, identify participants for the second phase of the study.

Thus, this quantitative research phase determined whether the expected differences in teachers' mathematical beliefs existed within the sample of participants. These results then informed the second, qualitative research phase by identifying potential participants for personal interviews to further probe differences in beliefs and their influence on teacher practices related to the CCSSM.

Phase II: Qualitative Methods and Procedures

Following the explanatory mixed methods design described by Creswell (2009), the quantitative data gathered from the questionnaire and MBS identified six participants for interviews to gather qualitative data to better “understand the world from the subjects’ point of view, to unfold the meaning of their experience, to uncover their lived world” (Kvale & Brinkmann, 2009, p. 1). This qualitative data, gathered in the second research phase, addressed the research question:

- 2) *How do these beliefs influence the ways teachers interpret and implement the Common Core State Standards for Mathematics?*

Participants and Selection Process

Summary statistics from the quantitative data informed the stratified purposeful sampling in the qualitative phase. Stratified purposeful sampling is defined as “the targeted selection from a stratified sample” (McMillan & Schumacher, 2010). Same grade teachers with the greatest range of mean scores were targeted for selection for personal interviews. Thus, the sample for the second research phase consisted of six (n=6) teachers: three pairs of same-grade teachers. The grade level of each participant pair was determined by the grade levels that offered the most numerically diverse teacher mean scores from MBS responses. Each same-grade pair consisted of teachers who hold different mathematical beliefs as measured by the MBS; one teacher earned the minimum overall mean score while the other teacher earned the maximum overall mean score for the particular grade level.

The purpose of the qualitative research phase in this study was to generate a rich understanding of each teacher’s experience with the phenomenon—in this case, each teacher’s unique interpretation and implementation of a CCSSM mathematics standard. A sample size of six interview participants was appropriate for the purposes of this study. The qualitative data generated in the interviews allowed a more comprehensive explanation of how differences in mathematical beliefs translate into differences in the ways teachers interpret and implement the CCSSM. Since the study was designed to explain, the findings produced here may or may not be generalizeable to the entire sample in this study or to the larger population of mathematics teachers. However, the findings can be used as a starting point for examining the intersection of teacher beliefs,

their practices, and the CCSSM from which additional research can build. The interview method allowed for an in-depth examination of each participant's beliefs and individualized experiences interpreting and implementing specific CCSSM mathematics standards. As each interview was conducted and transcribed by the researcher, the sample size took into consideration the resources available for the study and the need to generate a rich data set from which to answer the second research question.

Sample. As in the larger sample, the six teachers interviewed were responsible for teaching mathematics to students in a state that adopted the CCSSM in 2010. This sample (n=6) was comprised of the lowest and highest scoring teacher participants from the three grade levels that reflected the largest range in raw and overall mean scores.

Participant Protection. The participants selected for the interview research phase signed the original informed consent letter, indicating their willingness to voluntarily participate in an interview, if selected. To protect the identities of the teacher participants, only the researcher conducted the interviews, which were audio-recorded. The researcher used the three-digit identification number for each participant on the recording. During the analysis portion of the second research phase, the six participants were referenced using a new identification code comprised of a single digit number (grade level) followed by either LC (lower constructivist) or HC (higher constructivist).

Qualitative Instruments

Interview. Qualitative interviews “are particularly well suited for studying people’s understanding of the meanings in their lived world, describing their experiences and self-understanding, and clarifying and elaborating their own perspective on their

lived world” (Kvale and Brinkmann, 2009, p.116). The purpose of these qualitative interviews was to have conversations to uncover and analyze six teachers’ views and experiences with interpreting and implementing specific mathematics standards. These interviews took place in each teacher’s classroom or school conference room in March or April, 2015.

Each of these interviews utilized a semi-structured life world interview, defined as “an interview with the purpose of obtaining descriptions of the life world of the interviewee in order to interpret the meaning of the described phenomena” (Kvale and Brinkmann, 2009, p. 3). The interview protocol was designed using a script comprised of seven pre-determined questions related to one grade-level mathematics standards from the CCSSM. (See Appendix C for the complete set of interview questions and CCSSM standards reference for Grades 2, 3, and 4). The researcher asked questions in an established order and each interview participant responded in a conversation lasting approximately 20 minutes. The semi-structured design, permitting both follow-up questions and prompts, provides flexibility within each interview. To increase the quality of knowledge produced in the interview, the flexible semi-structured format allows researchers to clarify participant responses and construct meaning during the interview process (Kvale & Brinkmann, 2009).

Each semi-structured interview followed the interview script (see Appendix C) designed specifically for each grade level. A section of the introduction to the grade level CCSSM content standards and one specific mathematics standard was printed for the participant to use as a reference during the interview. Participants also used the paper to

write or draw to support their responses. (Any writings or drawings were collected by the researcher.) Questions were asked in order, but the semi-structured format allowed the researcher to ask for clarification, elaboration, or revisiting of earlier questions. To gather data that would lend itself to concept analysis and probe the participants' mathematical beliefs further, the researcher organized interview questions about interpretation and implementation practices related to the three belief-factor framework used in the study:

- Beliefs About Mathematics—whether skills should be taught in relation to conceptual understanding and problem solving and the order in which mathematical topics should be taught and learned
- Beliefs About Learning Mathematics—how children learn mathematics
- Beliefs About Teaching Mathematics—how mathematics should be taught

For example, the CCSSM calls for fourth grade students to “develop fluency with efficient procedures for multiplying whole numbers; understand and explain why the procedures work based on place value and properties of operations; and use them to solve problems” (NGA & CCSSO, 2010, p. 27). In Grade 4, Number and Operations in Base Ten (4.NBT), the fifth standard requires students to

Multiply a whole number of up to four digits by a one-digit whole number, and multiply two two-digit numbers, using strategies based on place value and the properties of operations. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models. (NGA & CCSSO, 2010, p. 29)

Interview questions for this grade level addressed each participant's interpretation of the language of the expectation and the standard:

- 1) What does the word *fluency* mean to you? How does a student demonstrate *fluency*?
- 2) Placed together in the standard, what do the words *efficient procedures for multiplying whole numbers* mean to you?
 - a) Describe what the word *efficient* means to you? What makes a method *efficient*?
 - b) What does the word *procedures* mean to you? Why is the word plural?

Subsequent questions focused on the three belief factors:

- 3) What prior skills and understanding do your students need to multiply two two-digit numbers?
- 4) What strategies do your students use to multiply two two-digit numbers? How do your students learn these strategies?
- 5) Describe how students demonstrate their skills and understanding of multiplying two two-digit numbers?
- 6) What is the relationship among skills (basic multiplication facts and computation procedures), conceptual understanding (what multiplication is), and problem solving?
 - a) Should children master multiplication facts/computation procedures *before* they solve word problems involving these skills or can experiences solving such problems help children learn these skills?
 - b) Can children who have not mastered basic multiplication facts/computational procedures solve word problems involving these skills? If so, how?
- 7) Describe your role in teaching students 4.NBT 5?

Aligning interview questions with the three belief factors measured by the MBS survey offered three key research benefits. First, the framework provided structure and focus for the participant's responses. Teaching and learning are complex; conversations about even one mathematics standard might have traveled in many directions that are interesting, but irrelevant to this study. Second, the framework provided the opportunity to create coding and categories during the analysis research stage. Third, aligning the constructs discussed in the interview with belief factors assessed in the survey allowed each participant to express his or her mathematical beliefs in more detail. The researcher was then able to use these detailed responses to support and extend the quantitative data set for each participant as well as compare data sets between participants.

Recording Devices. Interviews were recorded using the SuperNote program on an iPad device and a back-up recording was created using an Olympus mini-cassette recorder. As indicated, all interview recordings were stored in a secure location and destroyed after the completion of the study.

Interviewer. In qualitative research, the researcher is considered an instrument. To make participants more at ease, interviewers "should fit in as well as possible" (Fink, 2009, p. 39). Although the participants had been introduced to the researcher during the first research phase, to establish professional and collegial rapport with the participants, the researcher reintroduced herself as a teacher and doctoral student, restating that she has taught students, both mathematics and other content, at elementary and middle school grade levels in both traditional, intervention, and enrichment educational settings. The researcher also communicated that she has no previous teaching experience using the

CCSSM. This admission established her primary research role as that of a doctoral student wanting to learn more about how mathematical beliefs influence the ways teachers interpret and implement the CCSSM. The researcher is an experienced interviewer gained through other professional work, professional teaching background, and doctoral program coursework.

Qualitative Data Collection

Each interview will took place during March or April, 2015 in each participant's classroom or school conference room. The decision to use participants' classrooms for the interviews, when possible, was to use the familiar teaching environment to place each participant at ease and better enable him or her to recall interpreting and implementing the CCSSM as part of his or her professional practice. Each interview was scheduled for 30 minutes, with the flexibility to go longer if needed. Prior to the interview, participants were given a few minutes to read over the CCSSM reference sheet and interview questions (see Appendix C), asking questions if needed. The researcher began by asking the predetermined questions, in order, allowing participants to elaborate on his or her lived experiences with interpreting and implementing the CCSSM. The researcher asked for clarification or follow-up questions. Upon completion of the response to the final question, participants were invited to make any further comments specifically relating to mathematical beliefs and/or the processes of interpreting and implementing the CCSSM in the classroom. Once the participant had no further comments to share, the interview concluded and the researcher ended the audio recording. The researcher then shared the study protocol for transcribing, storing, organizing, analyzing, and destroying data.

Qualitative Data Organization and Analysis

In the second, qualitative research phase the raw data was textual—words in the language of the participants. Textual data is different than numerical data; therefore the analytical methods will also be different. Specific methods of analyzing data collected during qualitative interviews include coding, categorizing, and recognizing themes that emerge from the text. Coding involves “attaching one or more key words to a text segment in order to permit later identification of a statement” (Kvale & Brinkmann, 2009, pp. 201-202). Categorization, on the other hand, is a more “systematic conceptualization of a statement” (Kvale & Brinkmann, 2009, p. 202). The researcher uses codes as a means toward the goal of creating categories that accurately “capture the fullness of the experiences and actions studied” and data from participants can be “compared for similarities and differences” (Kvale & Brinkmann, 2009, p. 202). The study used concept-driven codes and categories based on the differences between constructivist and traditional theories of teaching and learning. The codes and categories directly related to the three belief factor framework used to analyze the quantitative survey data. (For the complete list of codes and categories see Appendix F.) The codes and categories within each interview text allowed themes to emerge, including determining whether responses aligned with a constructivist or non-constructivist perspective. These themes and determinations were, in turn, used to generate comparisons between the experiences of each participant.

To prepare the data for analysis, the researcher transcribed the recordings of the participant interviews to transfer the data from audio to text format. Line numbered

transcriptions created a word-for-word translation of the audio recording, including pauses, sighs, and other non-verbal vocalizations. All transcriptions were entered as a Microsoft 2007 Word document and later saved as a Portable Document Format (PDF) on the researcher's personal computer. Invented by Adobe, the PDF offers security features, such as password protection, that prevents others from accessing, copying, editing, or printing PDF documents (Adobe Systems, 2014). As with the quantitative materials collected in the first phase, all interview transcripts and audio tapes were to be destroyed after the successful defense of the dissertation, expected no later than September 1, 2015.

The researcher analyzed the qualitative data using a more general approach of interview analysis. The bricolage method is defined by Kvale & Brinkmann (2009) as “mixed technical and conceptual discourses where the interpreter moves freely between different analytic techniques and theories” (p. 323). In the bricolage method, the researcher reads the interview texts using theoretically informed interpretations. For the purposes of this study, the texts were read and interpreted to reflect each participant's mathematical views along the continuum from low-constructivist (or traditional) to high-constructivist, using tenets of constructivist learning theory as described in chapter two. Each participant's responses were interpreted as further explanation of his or her survey responses, again using the three belief-factor framework. This eclectic method of analysis (Kvale & Brinkmann, 2009) allows the researcher to move beyond codes and categories to construct meaning for participant views by incorporating “knowledge of the subject matter of analysis” in addition to “specific analytic techniques” (p. 233).

Theoretically, methodologically, and philosophically, this analytical approach answered Cobb's (2007) call for mathematics education researchers to become bricoleurs (p. 29) to reconcile philosophical differences. Bricoleurs construct using whatever tools are available; researchers as bricoleurs need not be constrained by choosing a single approach, but instead may build a comprehensive understanding of using multiple perspectives and approaches. As Cobb (2007) indicates, mathematics is a "complex human activity" (p. 29). It follows that teaching students about mathematics is also complex. Mathematics teaching and learning can be studied from a singular philosophical lens, such as experimental psychology, cognitive science, sociocultural theory, or distributed cognition theory; it can also be studied from a framework that uses more than one lens to view a complex construct (Cobb, 2007). As a bricoleur, this mathematics education researcher sought to understand teacher beliefs and practices via an eclectic analysis of data that drew from a combination of ethnographic (teaching is a social and cultural activity), phenomenological (the CCSSM is a phenomenon teachers experience), and case study (each teacher's unique practices associated with the CCSSM) research approaches.

To establish support for the interpretations of participant responses, the researcher analyzed the transcripts using a system of codes based on the words and meaning of the interviewee's responses. According to Kvale and Brinkman (2009), "coding involves attaching one or more keywords to a text segment in order to permit later identification of a statement," (pp. 201-202). The coding used in this qualitative interview data analysis was data driven, meaning that "the researcher starts out without codes, and develops them

through readings of the material” (Kvale and Brinkmann, 2009, p. 202). After several readings of the transcriptions, opportunities for coding emerged from the transcribed pages.

The researcher employed the methods and procedures previously described to analyze the content of each participant’s interview data set. Further analysis of the MBS responses led to additional interpretations of meaning related to the mathematical beliefs of each participant. These two sets of interpretations, gathered from the MBS survey responses and interview texts, were then combined into a new data set which was analyzed to construct themes for each individual participant.

The researcher organized these six new data sets into individual teacher profiles, by grade level, organized according to the interview questions related to interpretation and implementation. A comparative analysis between same-grade teacher pairs was followed by a comparative analysis between the low-constructivist teachers and the high-constructivist teachers. The researcher used this belief-based analysis to construct universal themes (thematic analysis) that allowed comparisons between the two groups and express how the differences in mathematical beliefs of teachers are related to differences in their practices of interpreting and implementing the CCSSM.

Qualitative Research Question and Expected Findings

The sequential explanatory design in this study addressed two related research questions. In the qualitative phase of research, data was collected to answer the second research question:

2) *How do these beliefs influence their interpretation and implementation of the Common Core State Standards for Mathematics?*

Although the CCSSM offers a standardized approach to what K-12 mathematics students are expected to know and be able to do at various points in their academic careers, the CCSSM dictates neither curriculum nor pedagogy. Each standard presented in the CCSSM, therefore, leaves room for teachers to use curriculum materials and instructional approaches that fit their interpretations and classroom vision for implementing the standards. Therefore, while the CCSSM document offers a common set of mathematics standards, the reality of the mathematics classroom experience may not be as common.

If beliefs are the best indicators of actions and decisions (Dewey, 1910), it follows that teachers who hold sharply different views of mathematics, teaching mathematics, and learning mathematics will teach differently—even when teaching the same standard. Beliefs, as “lenses that affect one’s view of some aspect of the world or as dispositions toward action” (Philipp, 2007, p. 259), also act as a filter through which new phenomena are interpreted (Pajares, 1992), including the phenomena of the CCSSM. As lenses, beliefs determine how teachers *interpret* the CCSSM; as dispositions toward action, beliefs influence how teachers *implement* the CCSSM.

The researcher expected that teachers who hold different mathematical beliefs, as indicated by overall mean scores and responses to items on the MBS survey, would likely interpret and implement the same mathematics standards differently. Additionally, the researcher expected that there might be similarities in how different-grade level teachers who hold similar mathematical beliefs interpret and implement mathematics standards

from the CCSSM across grade levels. The qualitative data gathered during the interviews revealed insight into the ways teachers' beliefs influence their practice.

Conclusion

Based on the theory of the powerful effect of beliefs, it follows that teacher beliefs about mathematics, teaching mathematics, and learning mathematics will act as a filter through which they interpret the CCSSM and, ultimately, implement these new mathematics standards in the classroom. The two questions posed by this study suggested a two-phase sequential explanatory mixed method research design. As beliefs are a complex and “messy” construct (Pajares, 1992), the use of a mixed methods design adhered to Cobb's (2007) suggestion that, to combat the limits inherent in using a single theoretical and methodological lens, researchers should instead act as bricoleurs—incorporating multiple lenses and methods to gather the best kind of data to answer the question(s) asked in the study.

In the first phase, the researcher used a pre-existing survey instrument to gather quantitative data from a larger sample of teacher participants to test the theory that not all teachers hold identical beliefs about mathematics, teaching mathematics, and learning mathematics. Statistical analysis of this numerical data informed the second phase by identifying six teacher participants for personal interviews. The six participants included three same-grade teacher pairs; one low-constructivist and one high-constructivist, as assessed by the survey. In the second research phase, the researcher conduct semi-structured interviews to further probe participants' mathematical beliefs and the ways they interpret and implement a specific CCSSM mathematics standards. The qualitative

interview data sets, textual data in participants' own words, were analyzed for content to allow categories and themes to emerge. These individual data sets were then combined so that universal themes could be constructed to better understand and explain the complex relationship between teachers' mathematical beliefs and their professional practices related to the CCSSM.

The purpose of this study was to understand what teachers believe about the nature of mathematics, how it should be taught, and how it is learned as well as how these beliefs shape the way they interpret new information—such as mathematics standards—and the actions they take in the classroom to implement those new standards. The study was descriptive research, in the sense that it not only described how teachers interpret and implement educational policy—policy designed to offer consistency in mathematics education across the United States—but also explained why differences in mathematical beliefs may account for differences in teaching practice. The following chapter is a report of results from both the MBS survey as well as the six personal interviews that shed light on the mathematical beliefs teachers hold and how those beliefs shape their professional decisions and actions, related to the CCSSM, in the classroom.

CHAPTER FOUR: DATA ANALYSIS AND RESULTS

Introduction

As described in this study, beliefs are lenses that affect one's world view or dispositions toward action (Philipp, 2007) and serve as a filter through which new phenomena are interpreted (Pajares, 1992). Thus, beliefs are lenses through which teachers *interpret* the Common Core State Standards for Mathematics (CCSSM) (NGA & CCSSO, 2010) and beliefs are dispositions toward action which affect how teachers *implement* the CCSSM. This study focused on two research questions:

- 1) *What do teachers believe about mathematics, teaching mathematics, and learning mathematics?*
- 2) *How do these teacher beliefs influence their interpretation and implementation of the Common Core State Standards for Mathematics?*

Study Context

The study was set in five schools in a public, mid-Atlantic, regional school district serving students in kindergarten through eighth grade. Data collection occurred at three of the four elementary (K-4) schools and the intermediate (5-6) school, involving teachers serving students in kindergarten through sixth grade. Quantitative data was collected using a demographic questionnaire and the Mathematics Beliefs Scales (MBS) (Fennema et al., 1990) survey instrument; qualitative data was collected during personal interviews with six teacher participants. This chapter reports on the analysis and results of both types of data to better understand teachers' mathematical beliefs (concerning the

first question) and how those beliefs influence professional practice (concerning the second question) in the classroom.

Phase I: Quantitative Data Analysis and Findings

Introduction

In the first phase of the study, participants completed the MBS, a Likert-style survey instrument designed to measure what teachers believe about mathematics, how teachers should teach it, and how students best learn it. The researcher administered the MBS survey to teachers responsible for teaching mathematics to students in any educational setting (n=80). The researcher converted responses to the 48 survey items to numeric values and organized the data by grade level. Analyses of the survey data from the first research phase are presented in five sections. The first three sections relate to the quantitative data analysis used to identify and categorize *participants* for the second research phase.

- Section 1 describes the statistical data analysis of teacher beliefs—particularly raw scores, overall means scores, and ranges in scores—used to identify same-grade teacher pairs (n=16), the lowest and highest scoring teachers from each of the eight grade levels.
- Section 2 describes how these 16 teacher participants were placed on a continuum from lowest possible score (48) to highest possible score (240) by overall mean score, enabling the researcher to categorize each participant according to the degree to which his or her beliefs aligned with the constructivist learning theory as described in chapters two and three.

- Section 3 discusses the selection of three same-grade teacher pairs ($n=6$) for one-on-one semi-structured interviews.

The researcher also utilized the survey data to inform the *content* of the second research phase.

- Section 4 explains how the researcher used item analysis to identify statements on the MBS survey that yielded the greatest differences in response values between same-grade teacher pairs.
- Section 5 describes how the meaning or language of these survey items that yielded the greatest differences in response was used to craft the final version of interview questions, enabling the researcher to further explore differences in beliefs within the context of practices related to a single mathematics standard.

Section 1: Teachers' Mathematical Beliefs

To answer the first research question, data generated from the MBS survey was used to identify what teachers believe about mathematics, teaching mathematics, and learning mathematics. This data also highlights differences in beliefs within a sample of teachers ($n=80$) for which several variables that influence practice—such as state or district standards, professional development and training, curriculum, classroom materials, classroom demographics—were common across the sample.

Participation and Demographic Data. Of the 83 teachers eligible to participate in the study, 80 returned signed informed consent letters and answered the questionnaire and survey items completely. The researcher used all 80 teacher participants' responses

in the subsequent analysis. The response rate for elementary teachers (K-4) was 100% and for intermediate teachers (5-6) was 85%, with an overall response rate of 96.39%.

The purpose of gathering demographic information in addition to responses on the MBS survey was to identify the grade level taught by each teacher participant and aid in the selection of interview participants, if required. All participant data was organized by grade level based on demographic questionnaire responses (see Table 2). No other demographic data was used in the analysis.

Table 2

Teacher Participants by Grade Level

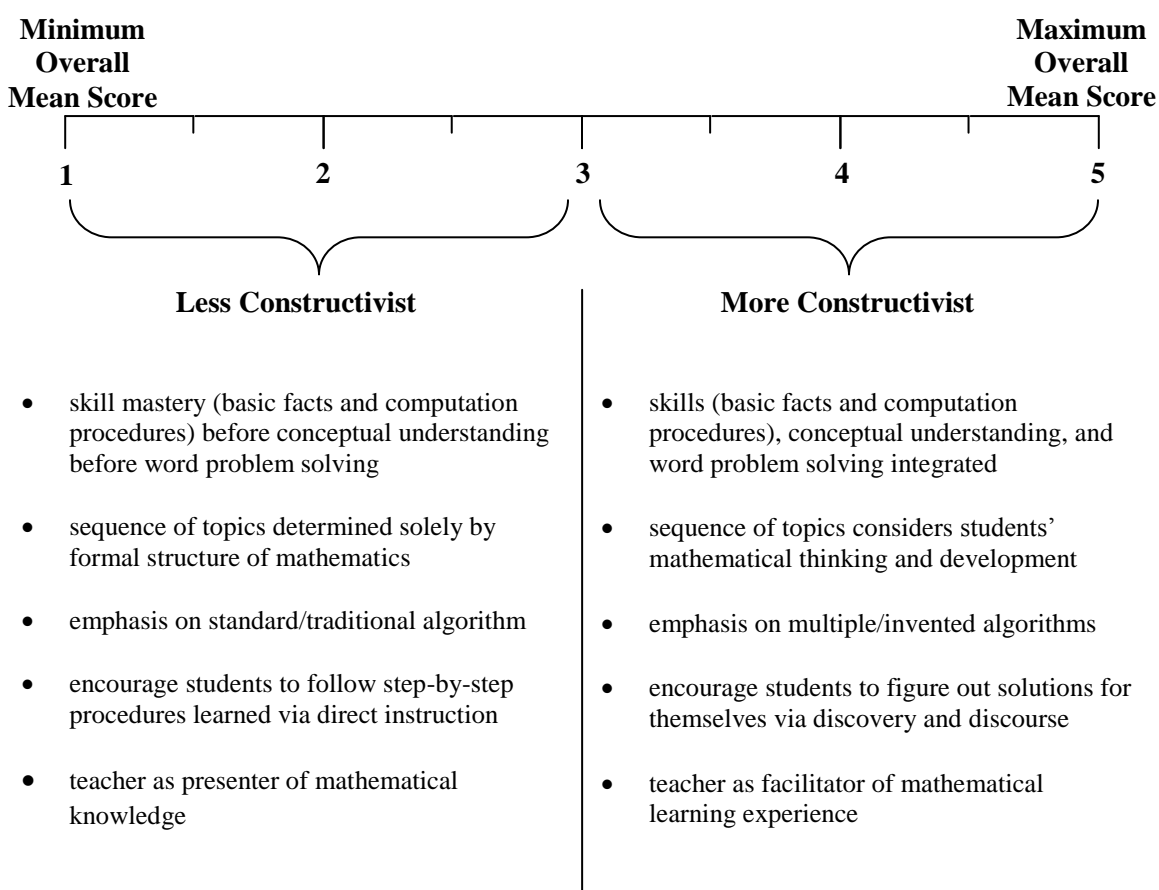
Grade Level	Number of Participants	Percent of Total Participants
Kindergarten	7	8.75
Grade 1	13	16.25
Grade 2	17	21.25
Grade 3	9	11.25
Grade 4	8	10.00
Grade 5	7	8.75
Grade 6	10	12.50
Multiple Grades	9	11.25
Total	80	100.00

Section 2: Analyzing Teacher Beliefs. The MBS survey is designed using a five-point Likert-type scale where a score of 3, used to indicate an ‘undecided’ response, is

considered neutral. Response values of 4 and 5 indicate more constructivist beliefs while response values of 2 or 1 indicate less constructivist, or traditional, beliefs. (See Figure 2).

Figure 2

Mathematical Beliefs Continuum



Section 3: Selecting Interview Participants. Each participant's responses were totaled to reflect a raw score; the 48-item survey having a minimum overall possible score of 48 and a maximum overall possible score of 240. The researcher then calculated an overall (arithmetic) mean score for each participant in the sample using the formula

$$\bar{x} = \frac{\sum x}{n}$$

where $\sum x$ is the sum of all data values
and n is the number of data items in the sample

The results of all survey data for individual participants, showing grade-level minimum, maximum, and ranges of overall mean scores, are summarized in Table 3. The raw score and overall mean score for each participant was calculated to compare individual data to the mean raw score and average overall mean score to other same-grade participants. (See Appendix E for additional grade level data for low- and high-scoring participants, raw score mean, overall mean score mean, and standard deviation.)

Table 3

Descriptive Statistics by Grade Level

Grade Level	Minimum Raw Score	Maximum Raw Score	Range in Raw Score	Minimum OMS	Maximum OMS	Range in OMS
Kindergarten	141	194	53	2.94	4.04	1.10
Grade 1	149	191	42	3.10	3.98	0.88
Grade 2	141	202	61	2.94	4.21	1.27
Grade 3	131	193	62	2.73	4.02	1.29
Grade 4	138	209	71	2.88	4.35	1.47
Grade 5	156	212	56	3.25	4.42	1.17
Grade 6	149	205	56	3.10	4.27	1.17
Multiple Grades	152	210	58	3.17	4.38	1.21

Developing a Scoring Continuum and Participant Categories. This numerical data indicates there are differences in beliefs between same-grade teacher participants. The researcher created eight same-grade pairs (seven pair from grades K-6 and one pair of multi-grade teachers) by selecting the lowest and highest scoring participant from each grade level (n=16). Using overall mean scores, the researcher created categories and identification codes according to constructivist learning theory:

<u>OMS Score</u>	<u>Categorization</u>	<u>Code</u>
1.00 – 1.99	Very Low Constructivist	VLC
2.00 – 2.99	Low Constructivist	LC
3.00 – 3.99	Constructivist	C
4.00 – 4.99	High Constructivist	HC

Grades 2, 3, and 4 had the largest ranges in raw score, overall mean score, and categorization on the constructivist continuum—suggesting the strongest difference in mathematical beliefs between same-grade teachers. Results are summarized in Table 4.

Table 4

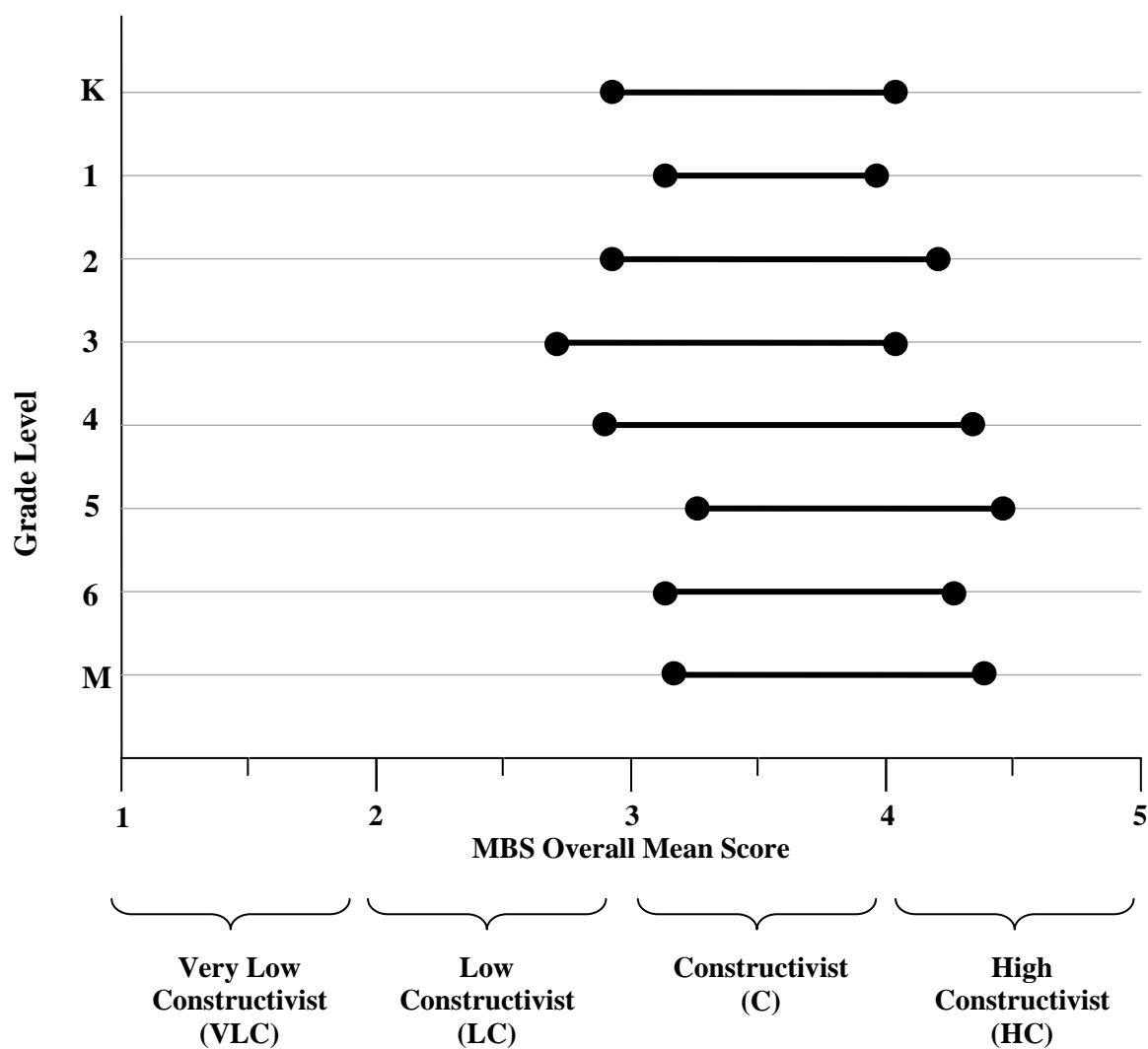
Same-Grade Teacher Pairs Data and Categorization

Grade Level	K	1	2	3	4	5	6	M
Maximum OMS	4.04	3.98	4.21	4.02	4.35	4.42	4.27	4.38
Categorization	HC	C	HC	HC	HC	HC	HC	HC
Participant ID#	224	236	257	159	151	102	170	158
Minimum OMS	2.94	3.10	2.94	2.73	2.88	3.25	3.10	3.17
Categorization	LC	C	LC	LC	LC	C	C	C
Participant ID#	220	230	254	124	154	172	105	142

The eight same-grade teacher pairs and their placement into categories on the constructivist continuum are shown in Figure 3.

Figure 3

Minimum and Maximum Overall Mean Scores by Grade Level



Identifying Six Interview Participants

The researcher selected the following same-grade teacher pairs, the lowest and highest scoring teacher participants from the grades reflecting the largest ranges in raw and mean scores as well as differences in categorization. At this point in the study, the researcher assigned interview participants a new identification code based on grade level and categorization:

Grade 2: 2LC = 254 and 2HC = 257

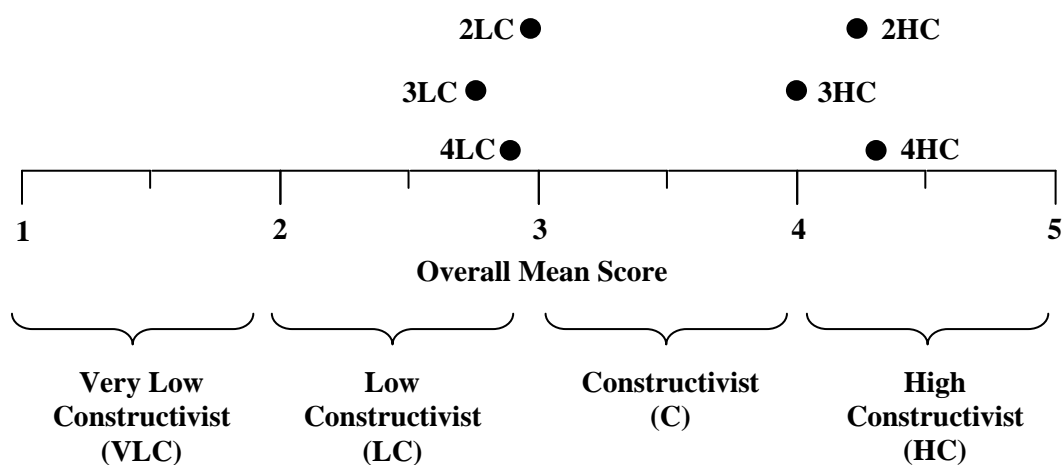
Grade 3: 3LC = 124 and 3HC = 159

Grade 4: 4LC = 154 and 4HC = 151

The placements of the six teachers along the constructivist continuum, which will be used frequently throughout the qualitative results section of the chapter, are shown in Figure 4.

Figure 4

Interview Participant Categorization by Overall Mean Score



Section 4: Identifying Focus Survey Items

In addition to using the quantitative data to select interview participants, the researcher conducted an item analysis to identify the survey items that generated the greatest differences in response between the two same-grade participants. There were no survey items for which same-grade teacher participants in Grades 2, 3, or 4 differed by 4 points—the maximum difference in a five-point Likert scale item. However, the researcher found 16 survey items on which same-grade participant responses differed by 3 points. The second grade teachers' responses differed by 3 points on 4 statements (Items 1, 16, 28, and 48). The third grade teachers' responses differed by 3 points on 7 statements (Items 10, 13, 28, 33, 37, 44, and 45). The fourth grade teachers' responses differed by 3 points on 11 statements (Items 1, 7, 10, 13, 14, 21, 22, 23, 28, 29, and 47). These 16 survey statements and respective point differences are shown in Table 5. (For the complete MBS survey, see Appendix B.)

The researcher then used comparative item analysis to identify survey items reflecting the greatest differences in response among the six teacher participants across all grade levels. Three survey items (shown in Table 6) generated a 3-point difference in response by teacher pairs across two grade levels. Cumulative differences were found by calculating the sum of the point differences for all three grade levels. The researcher identified five survey items (shown in Table 7) with a cumulative difference equal to 7 points. Finally, all three same-grade pairs reflected a 3-point difference on Item 28, for a cumulative difference equal to 9 points (as shown below in Table 8). These became focus items, shaping both interview questions and integrated data analysis.

Table 5

Item Analysis—Differences in Response Values

Item	Statement	Grade 2	Grade 3	Grade 4
1	Children should solve word problems before they master computational procedures.	3	0	3
7	The teacher should demonstrate how to solve simple word problems before children are allowed to solve word problems.	2	2	3
10	Even children who have not learned basic facts can have effective methods for solving problems.	1	3	3
13	Children should have many informal experiences solving simple word problems before they are expected to memorize number facts.	1	3	3
14	An effective teacher demonstrates the right way to do a word problem.	1	1	3
16	Most young children have to be shown how to solve simple word problems.	3	1	2
21	Children usually can figure out for themselves how to solve simple word problems.	2	2	3
22	Recall of number facts should precede the development of an understanding of the related operation (addition, subtraction, multiplication, or division).	2	2	3
23	Children will not understand an operation (addition, subtraction, multiplication, or division) until they have mastered some of the relevant number facts.	2	0	3
28	Children should be allowed to invent new ways to solve simple word problems before the teacher demonstrates how to solve them.	3	3	3
29	Time should be spent practicing computational procedures before children are expected to understand the procedures.	0	0	3
33	Children can figure out ways to solve many math problems without formal instruction.	2	3	0
37	Teachers should allow children to figure out their own ways to solve simple word problems.	1	3	0
44	It is important for a child to know how to follow directions to be a good problem solver.	2	3	1
45	To be successful in mathematics, a child must be a good listener.	2	3	0
48	Children learn mathematics best from teachers' demonstrations and explanations.	3	2	1

Table 6

Item Analysis—3-Point Response Difference in Two Grade Levels

Item	Statement	Grade 2	Grade 3	Grade 4
1	Children should solve word problems before they master computational procedures.	3	0	3
10	Even children who have not learned basic facts can have effective methods for solving problems.	1	3	3
13	Children should have many informal experiences solving simple word problems before they are expected to memorize number facts.	1	3	3

Table 7

Item Analysis—7-Point Cumulative Difference

Item	Statement	Grade 2	Grade 3	Grade 4
7	The teacher should demonstrate how to solve simple word problems before children are allowed to solve word problems.	2	2	3
10	Even children who have not learned basic facts can have effective methods for solving problems.	1	3	3
13	Children should have many informal experiences solving simple word problems before they are expected to memorize number facts.	1	3	3
21	Children usually can figure out for themselves how to solve simple word problems.	2	2	3
22	Recall of number facts should precede the development of an understanding of the related operation (addition, subtraction, multiplication, or division).	2	2	3

Table 8

Item Analysis—9-Point Cumulative Difference

Item	Statement	Grade 2	Grade 3	Grade 4
28	Children should be allowed to invent new ways to solve simple word problems before the teacher demonstrates how to solve them.	3	3	3

Section 5: Using Focus Items to Inform Interviews and Qualitative Data Analysis

Identifying and analyzing items answered most differently informed the final wording of the interview questions and content for follow-up questions during the second phase of the study, allowing for a deeper, focused probe into how differences in teacher beliefs play out in the classroom environment. Additionally, pairing such items from the survey with one specific grade-level mathematics standard from the CCSSM enabled the participant to explain, in his or her own words, personal meaning of the standard as well as individual professional decisions and actions related to implementing that same mathematics standard. Focus item data was also used during the second research phase at the individual level to compare each teacher's interview responses to survey items with which he or she strongly agreed or disagreed. Thus, the pairing of focus items and a specific standard provided each teacher participant and the researcher a context to further explain beliefs measured by the survey within the real world setting of the mathematics classroom.

Phase I Summary

The numerical data collected using the MBS survey not only determined the six teacher participants selected for personal interviews, but also identified items (1, 7, 10, 13, 21, 22, and 28) reflecting the greatest differences in beliefs for all three teacher pairs. These items informed the crafting of both initial and follow-up interview questions in the second phase of research. Specific survey item responses for individuals were used in combination with qualitative responses during the analysis phase. Following Creswell's (2009) explanatory mixed methods design, the researcher was able to use the first-phase,

quantitative data from a larger sample of 80 participants to inform the selection of a smaller sample of participants, the specific content for second-phase qualitative interviews, and analysis of that interview data. Thus, the second research phase was designed to collect detailed views about teachers' beliefs and practices related to the CCSSM within the practical world of the classroom to further explain their quantitative responses to theoretical survey questions.

Phase II: Qualitative Data Analysis and Findings

Introduction

To better understand the first-phase survey results and answer the second research question concerning whether and how differences in beliefs influence teacher practices, the researcher collected qualitative data by conducting personal interviews with six teacher participants (2LC, 2HC, 3LC, 3HC, 4LC, and 4HC) from March 30 through April 24, 2015. These interviews enabled the researcher to construct a better understanding of the complex beliefs-practice relationship within the context of a specific CCSSM mathematics standard. The semi-structured interview protocol asked seven core questions and four sub-questions to uncover each participant's meaning of the standard's language as well as the ways to teach and learn the mathematical content to meet that standard. Interviews lasted between sixteen and twenty-four minutes, depending on the teacher, the detail of responses, and the number of follow-up questions asked.

After transcribing the interviews, the researcher analyzed each transcript to generate codes and categories using an iterative review process. Characteristics of the constructivist learning theory and traditional versus reform mathematics education

models (described in the second chapter) were used to classify practices as less constructivist or more constructivist. The researcher placed comments describing practices that were open to interpretation in both categories and addressed possible interpretations using related responses to provide context. Using this analysis approach, the researcher coded specific parts of each response to provide a broad characterization of the response as a whole.

Several themes emerged from the qualitative data:

- the role of basic facts in developing mathematical knowledge
- the role of word problem solving in developing mathematical knowledge
- the importance of and practical uses for mathematics in the real world
- the use of the standard algorithm or alternate/multiple algorithms
- the importance of understanding the mathematical reasoning behind procedural steps in any algorithm
- the use of teacher demonstration versus discovery learning approaches

These themes were organized to align with the mathematics belief three-factor framework adapted from Fennema et al. (1990), Capraro (2001), and Ambrose (2004) that relate to the three types of mathematics beliefs studied in the quantitative research phase:

- Implementation Practices Based on Beliefs About Mathematics (the relationship among skills, understanding, and word problem solving; the sequencing of mathematics topics)
- Implementation Practices Based on Beliefs About Learning Mathematics

- Implementation Practices Based on Beliefs About Teaching Mathematics

Interview Participants

With one exception, all interview participants had between 11 and 18 years of teaching experience. Teacher 3LC, however, was in the first year of teaching. To ensure this interviewee's responses accurately reflected the beliefs-practice relationship for the purposes of this study, the researcher explained that responses could relate to realistic (current) or idealistic classroom practices and students to reduce the influence of classroom management issues and other factors that may affect the implementation of academic standards.

To meet the overarching goal of mathematics education, to ensure all children become mathematically proficient, the National Research Council (NRC) (2001) recommended an integrated and balanced approach that included:

- 1) conceptual understanding
- 2) procedural fluency
- 3) strategic competence
- 4) adaptive reasoning
- 5) productive disposition (p. 11)

The NRC also warned against extreme instructional positions that “concentrate on one strand of proficiency to the exclusion of the rest” (2001, p. 11). None of the three LC teachers had scores categorized as very low constructivist (VLC) on the continuum. Each had overall means scores near the neutral score of three: 2LC=2.94, 3LC=2.73, and 4LC=2.88. 2HC, 3HC, and 4HC had overall mean scores of 4.21, 4.02, and 4.35,

respectively. Since none of the teachers' scores are at the extreme end of the continuum, it was expected that a participant's responses might include both low and high constructivist practices. This is particularly relevant for the LC teachers whose scores were near a neutral score of 3 on the continuum. The researcher used specific item analysis, related to particular mathematics beliefs, to select focus items specific to the group as well as individual teachers to gain a better understanding of whether certain practices are more or less consistent with the tenets of constructivism. The intent was to determine whether the consonance or dissonance of teachers' beliefs, between one another and between an individual teacher and the beliefs that underlie the CCSSM, would translate into one mathematics standard looking different when placed in the hands of each teacher.

To examine the filtering effects of beliefs on the ways one interprets new phenomenon (Pajares, 1992), data gathered in the first part of each qualitative interview focused on the ways each teacher interprets specific words and phrases taken directly from the CCSSM grade-level introduction section and one mathematics standard. To better understand how beliefs serve as dispositions toward action (Dewey, 1910), data collected in the second part of the interview focused on the ways each teacher implements the standard through various classroom practices.

Qualitative analysis and results are organized into two main sections. The first section serves to orient the reader by providing an overview of key findings. The second section presents detailed integrated data analyses (including quantitative and qualitative data) that support those findings: individual teacher profiles, within grade-level

comparative analysis, and comparative analysis between LC and HC teachers. While the researcher used verbatim transcriptions of each interview, some of the direct quotes presented in chapters four and five have been adjusted for readability purposes including the removal of fillers and minor grammatical corrections, as suggested by Kvale and Brinkmann (2009, pp. 186-187).

Section 1: Overview of Qualitative Findings

Both similarities and differences existed in the meanings teachers constructed for specific language used in the CCSSM, as depicted in both their direct interpretation of the standard and described classroom environment relating to that interpretation. Regarding implementation, LC teachers described classroom practices that were both constructivist and non-constructivist in nature, typically trending toward the traditional (non-constructivist) end of the continuum and aligning with their Low Constructivist (LC) categorization based on overall mean survey scores. Survey item analysis found LC teachers expressed conflicting beliefs on the survey and offered more *neutral* responses and fewer *strongly (dis)agree* responses. By contrast, HC teachers described implementation practices that were nearly all constructivist in nature, typically trending toward the high constructivist end of the continuum and aligning with their High Constructivist (HC) categorization based on overall mean survey scores. Survey item analysis found HC teachers expressed consistently constructivist beliefs on the survey and offered fewer *neutral* responses and more *strongly (dis)agree* responses.

CCSSM Interpretation and Mathematical Beliefs. All teachers' initial interpretations of the word fluency were similar, but means of achieving fluency varied

by teacher. LC teachers equated the word efficient with one particular calculation method—the traditional method (or standard algorithm). To HC teachers, the word efficiency meant that a student is comfortable and proficient with whatever method he or she chooses to use. All teachers interpreted the plural use of the word *methods* (Grade 2), *strategies* (Grade 3), or *procedures* (Grade 4) to mean that there are different ways for students to solve mathematics problems.

CCSSM Implementation and Beliefs about Mathematics. LC teachers described both constructivist and traditional implementation practices, including an emphasis on basic facts before procedures and word problems. HC teachers described consistently constructivist implementation practices involving the integration of skills, understanding, and word problem solving.

CCSSM Implementation and Beliefs about Learning Mathematics. LC teachers described both constructivist and traditional implementation practices, including the need for students to solve calculation problems using the prescribed, step-by-step, traditional method (Grades 2 and 4) or the view that using models indicates a lower level of thinking than using arithmetic to solve word problems (Grade 3). HC teachers described highly constructivist implementation practices including inquiry-based learning, guided discovery, discussion of mathematical ideas, emphasis on multiple and/or alternate algorithms, and the use of manipulatives to help students learn facts, understand computational procedures, and solve word problems.

CCSSM Implementation and Beliefs about Teaching Mathematics. LC teachers described mostly traditional implementation practices, including the teaching-as-

telling instructional model (Smith, 1996), a teacher-created anchor chart to demonstrate solutions (Grade 2), the desire to help students avoid struggle while learning (Grade 3), or emphasis on basic fact drills and rigorous practice of the traditional method (Grade 4). HC teachers described highly constructivist implementation practices including inquiry-based teaching, guided discovery, encouraging multiple voices in the mathematics classroom, eliciting student thinking to guide instruction, and helping students see connections (between actions and procedures or between operations).

Section 2: Individual and Comparative Data Analysis

Individual Teacher Profiles: Grade 2. The Grade 2 CCSSM standard selected for this study involves the use of place value understanding and properties of operations to add and subtract numbers within 1000 (NGA & CCSSO, 2010).

The first two interview questions encouraged teachers to explain the meaning of the word “fluency” and the phrase “efficient, accurate, and generalizable methods” in the CCSSM Grade 2 Introduction section (p. 18) and Numbers and Operations in Base Ten, Standard 7 (NGA & CCSSO, 2010, p. 19).

The remaining interview questions asked teachers to describe implementation practices related to the three belief factors in the study: Beliefs about Mathematics (the relationship among skills, understanding, and word problem solving; the sequencing of mathematics topics), Beliefs about Learning Mathematics, and Beliefs about Teaching Mathematics. Two questions (related to beliefs about mathematics) asked what prior skills and understanding students need to add and subtract within 1000 and how skills (basic facts and computational procedures), conceptual understanding of both operations,

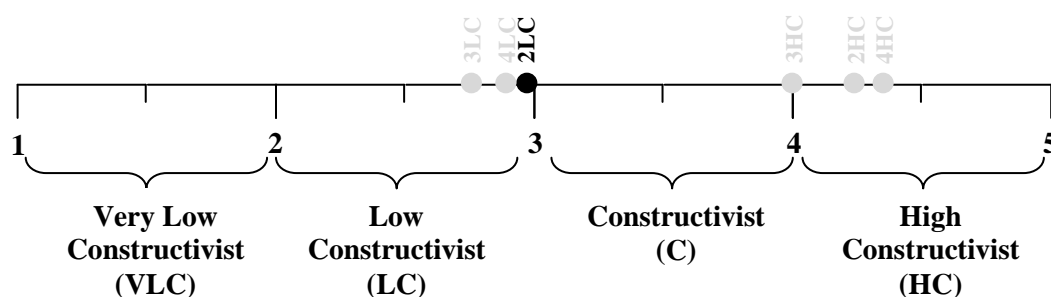
and word problem solving are related. Sub-questions specifically asked whether students should master skills before word problem solving. Two questions (related to beliefs about learning mathematics) asked teachers to describe the strategies students use to add and subtract within 1000, how students learn those strategies, and how students demonstrate their learning of the standard. The final interview question (related to beliefs about teaching mathematics) asked teachers to describe their role in helping students meet the standard. (See Appendix C for complete sets of Grades 2, 3, and 4 interview questions.)

Results for each of the second grade teachers include a teacher profile, interpretation results, and implementation results organized by belief factor. The second grade section ends with a brief comparative analysis of teachers 2LC and 2HC.

Grade 2 Low Constructivist (Teacher 2LC) Profile. Teacher 2LC teaches second graders at elementary school 4 and has 11 years experience teaching in both the first and second grades. Teacher 2LC earned an overall mean score of 2.94 on the MBS survey, depicting low-constructivist beliefs (shown in Figure 5), but with a numerical score very close to the neutral score of 3.

Figure 5

Teacher 2LC: Placement on Constructivist Continuum



Teacher 2LC: CCSSM Interpretation. To open the interview, 2LC stated “A student demonstrates fluency by being able to add and subtract smoothly, quickly, and comfortably without any hesitation.” This teacher did not list specific ways for a student to demonstrate fluency besides the ability to answer a question quickly and comfortably. When adding and subtracting numbers within 1000, 2LC said an efficient method involved students “being able to do it in their head using different parts, starting, you know, on the right side of the problem and moving on over.” This teacher commented that second graders may use fingers, a number line, or a calculator to add and subtract.

Teacher 2LC added that generalized methods for addition and subtraction within 1000 involved “being able to add and subtract simple numbers first and then going larger” by using regrouping. These interpretations of the language used in the standard (confirmed with comments regarding other interview questions) suggest that 2LC equates efficiency with *a* particular method, in this case the traditional method, or standard algorithm, of calculation by lining up numbers in place value columns, starting with ones place, and regrouping when necessary. These interpretations, particularly that students should know basic facts “in their head” and use the standard algorithm when adding and subtracting numbers within 1000, are aligned with less-constructivist mathematics beliefs indicted by an overall mean score of 2.94 as well as responses to specific survey items.

Teacher 2LC disagreed with the MBS statements:

Item 2: Teachers should encourage children to find their own solutions to math problems even if they are inefficient.

Item 33: *Children can figure out ways to solve many math problems without formal instruction.*

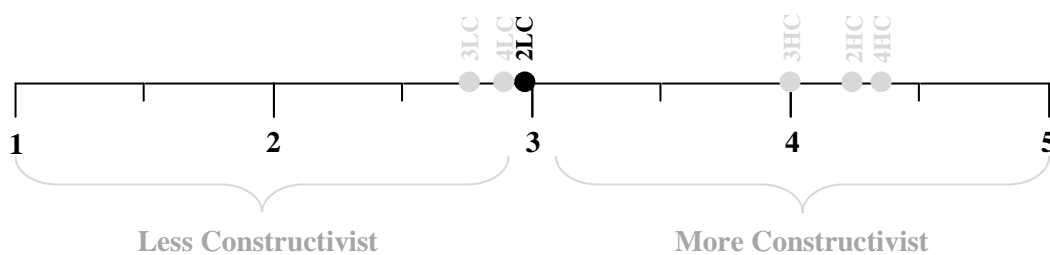
Teacher 2LC's practices of teaching students to use one particular (traditional) method for adding and subtracting within 1000 aligns with the less-constructivist beliefs indicated on the survey, reflecting a narrow interpretation for what constitutes an *efficient* calculation method.

Teacher 2LC: CCSSM Implementation.

Beliefs about Mathematics. Figure 6 provides a summary of 2LC's interview data related to this belief factor.

Figure 6

Teacher 2LC: CCSSM Implementation and Beliefs about Mathematics



2LC Implementation Practices

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|--|---|
| <ul style="list-style-type: none"> • basic facts and regrouping • “A lot of students may not have the experiences in life to solve the problems, so I think that’s why all students need to master addition and subtraction before solving word problems.” | <ul style="list-style-type: none"> • basic facts and regrouping • “understand why we regroup” • “needs to know not only the basic facts—basic addition and subtraction facts—but they also need to understand why they’re doing it” • “They need to master the skills, but I think when mastering the skills the experiences can help them do that.” • “I definitely think a student that has not, is still using fingers, is slow at doing it, can absolutely still solve word problems. It may just take some time doing it. But I definitely think they can.” |
|--|---|

Teacher 2LC identified prior knowledge of basic facts as well as a conceptual understanding of how and why to regroup in order to meet the standard. This notion of building new knowledge from prior learning is supported in a separate response: “being able to add and subtract simple numbers first and then going larger.” Teacher 2LC’s approach of using prior learning and making certain that students understand why and how to regroup suggest a more constructivist view of helping students see relationships between skills and understanding. When discussing word problems, 2LC continued this constructivist view, stating that students can use life experiences to help them master skills. 2LC then took a traditional turn: because not all students have such life experiences, 2LC concluded “that’s why all students need to master addition and subtraction before solving word problems.”

Teacher 2LC described implementation practices regarding the relationship among skills, understanding, and word problem solving that were both traditional and constructivist in nature. This aligns with 2LC’s overall mean score of 2.94, indicating beliefs that trend toward the middle of the continuum. Specific item analysis was used to determine whether 2LC offered neutral or contradictory responses within this belief factor.

Regarding the relationship between skills and understanding and the sequencing of mathematics topics, 2LC agreed with the following MBS statement:

Item 22: Recall of number facts should precede the development of an understanding of the related operation (addition, subtraction, multiplication, or division).

However, in a direct contradiction, 2LC also strongly agreed with this MBS statement:

Item 6: *Children should understand the meaning of an operation (addition, subtraction, multiplication, or division) before they memorize number facts.*

Teacher 2LC thus expressed opposing beliefs about whether children should master facts before developing a conceptual understanding of the related operation.

Questions involving word problem solving and computational procedures reflected inconsistencies as well. Teacher 2LC agreed with this MBS statement:

Item 4: *Time should be spent solving simple word problems before children spend much time practicing computational procedures.*

Teacher 2LC, however, disagreed with the following MBS statement:

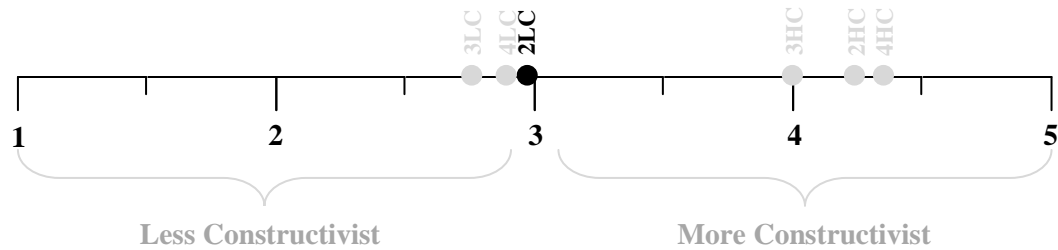
Item 2: *Children should solve word problems before they master computational procedures.*

Again, 2LC's beliefs appear to be contradictory. "A lot of students may not have the experiences in life to solve the problems, so I think that's why all students need to master addition and subtraction before solving word problems." Perhaps this comment reflects 2LC's belief that while it would be *ideal* for children to solve word problems first, the *reality* is that they lack the life experience to make this possible and therefore must practice computational procedures first.

Beliefs about Learning Mathematics. Figure 7 provides a summary of 2LC's interview data related to this belief factor.

Figure 7

Teacher 2LC: CCSSM Implementation and Beliefs about Learning Mathematics



2LC Implementation Practices

<ul style="list-style-type: none">• “I definitely teach them how to use a t-chart or a chart where they can have the hundreds, the tens, and the ones place. They always start at the ones place, I tell them. They subtract that. If they have they have to regroup, I teach how to regroup from the next place value.”• anchor charts: “where the teacher guides them first at the carpet with an anchor chart showing them how to do it”• “you model it and then you would hang it up in the classroom so the students can always go back to it and check to see ‘how did we do that again?’”• check work using inverse procedure	<ul style="list-style-type: none">• building onto “simple strategies of just adding and subtracting single digits”• using formative assessment to check for understanding• check work on their own using addition to check subtraction and using subtraction to check addition• sharing answers with partner(s)• using hands-on manipulatives• “anything that they can see visually and use with their hands is very helpful in introducing it”
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Teacher 2LC described implementation practices relating to how children best learn mathematics that are both traditional and constructivist in nature. This aligns with 2LC’s overall mean score of 2.94, indicating beliefs that trend slightly toward the traditional end of the spectrum. Specific item analysis was used to determine whether 2LC offered neutral or contradictory responses within this belief factor.

Regarding how children best learn mathematics, 2LC agreed with the following MBS statements:

Item 48: *Children learn mathematics best from teachers' demonstrations and explanations.*

Item 44: *It is important for a child to know how to follow direction to be good problem solver.*

These beliefs may influence the use of the anchor chart for demonstration purposes as well as the posting of the anchor so children can remember the way to solve that type of problem. Not only does this suggest that children need to follow directions, but encourages the view that the anchor chart helps student recall the 'right' way to solve the problem. Also students share their work with partners, it is clear that this teacher has shown them *the* way to solve such problems using the t-chart (place value) model. "I definitely teach them how to use a t-chart or a chart where they can have the hundreds, the tens, and the ones place. They always start at the ones place, I tell them." There is no mention of alternative methods to calculate sums and differences, such as partial sums, maintaining the difference, or a variety of other methods that students invent to solve problems.

These direct instruction practices extends to nearly all of 2LC's responses to survey items measuring beliefs about how children learn to solve word problems. 2LC strongly agreed with the following MBS statement:

Item 16: *Most young children have to be shown how to solve simple word problems.*

Yet, once again, 2LC exhibits conflicting beliefs by strongly agreeing with a statement proposing the exact opposite belief:

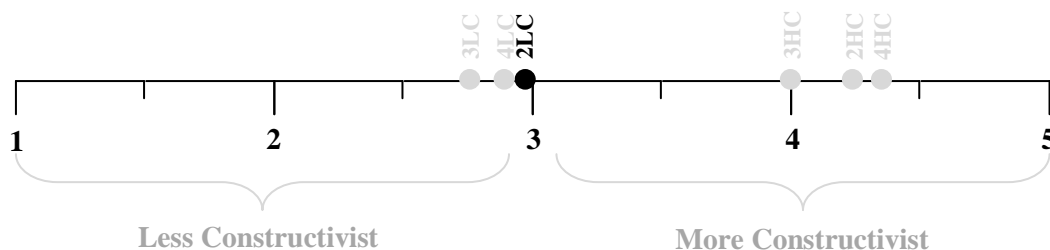
Item 20: *Children learn math best by figuring out for themselves the ways to find answers to simple word problems.*

When the teacher first demonstrates a particular problem solving method (whether computational or word) and hangs it on the wall to serve as a reference, children are no longer encouraged to figure things out for themselves and invent new ways to solve problems.

While most comments offered by 2LC could easily be interpreted as either less constructivist or more constructivist, the researcher placed one comment in both categories. Teacher 2LC described the practice of having students use inverse operations to check for accuracy—as directly called for in the standard. Encouraging students to find ways to check their own work and understand the relationship between inverse operations, such as addition and subtraction, could be considered a constructivist practice; mathematical authority is not limited to the teacher or answers in a textbook. However, if this method is presented as part of the procedural model—simply another “step” to follow—with little conceptual development of inverse operations, the practice reflects a traditional approach. Taken together, the classroom implementation practices for this mathematics standard strongly align with most survey item responses reflecting a less constructivist learning environment.

Beliefs about Teaching Mathematics. Figure 8 provides a summary of 2LC’s interview data related to this belief factor.

Figure 8

Teacher 2LC: CCSSM Implementation and Beliefs about Teaching Mathematics**2LC Implementation Practices**

- | | |
|---|---|
| <ul style="list-style-type: none"> • “I definitely teach them how” • “I tell them.” • teacher is “showing them how to do it” • “you model it” • “I have an important role in teaching that to them, probably the only role because they don’t get math anywhere else.” • teacher demonstration • anchor charts • ability grouping | <ul style="list-style-type: none"> • differentiated instruction • “students working in groups” • “students helping each other out” |
|---|---|

Teacher 2LC described some implementation practices that were constructivist in nature, such as students working in groups to help one another learn, but the majority trended toward a traditional model of teaching-as-telling. By stating that students “don’t get math anywhere else” combined with the numerous references throughout the interview to teacher modeling and demonstration, 2LC appears to view the teacher as the sole mathematical authority in the classroom. These described implementation practices to teach second graders how to add and subtract within 1000 reflect the show-and-tell beliefs associated with the traditional model of teacher efficacy (Smith, 1996). These beliefs are indicated by 2LC’s agreement with the following MBS statement:

Item 14: *An effective teacher demonstrates the right way to do a word problem.*

Again, this belief clearly influences this teacher's use of the anchor chart to show students the right way to solve a problem. However, 2LC's traditional, direct-instructional approach—aligned with most of the survey responses and classroom practices—is contradicted by his or her agreement with this MBS statement:

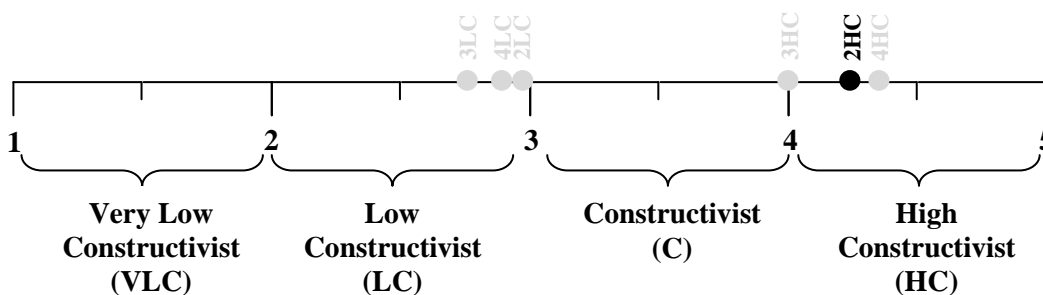
Item 9: *Mathematics should be presented to children in such a way that they can discover relationships for themselves.*

Within this belief factor, the majority of 2LC's beliefs and described implementation practices appear to reflect the sentiment behind the presentation portion of the previous MBS item much more than the sentiment behind the child discovery portion when teaching students how to add and subtract within 1000.

Grade 2 High Constructivist (Teacher 2HC) Profile. Teacher 2HC teaches second graders at elementary school 2 and has 18 years experience teaching first, second, and fifth grade students. Teacher 2HC earned an overall mean score of 4.21 on the MBS survey (shown in Figure 9), suggesting this teacher holds high-constructivist beliefs.

Figure 9

Teacher 2HC: Placement on Constructivist Continuum



Teacher 2HC: CCSSM Interpretation. Teacher 2HC stated that fluency “is the quickness and the smoothness...that a student can solve simple addition and subtraction problems and also mentally...their mental math skills, that fluency...the number sense that they understand” and “how quickly they can explain their thinking process.” For this teacher, fluency extends beyond basic fact recall to involve number sense (an understanding of number size, relationships, and operations) and mathematical reasoning.

Teacher 2HC described a variety of ways to demonstrate and assess fluency in the classroom:

Many ways...through games, and that is playing with me, playing with friends while I observe how they interact with them. Obviously, timed tests can express fluency. Describing how they would solve a problem, verbally, using their language: ‘I solved this because I carried this ten or I know that if I break up this number I can find a complement of ten.’ Using their mental math skills...verbally demonstrating it in games, demonstrating it in paper and pencil, and also when they can teach another student. I think that also helps demonstrate, I think, at a higher level what they truly understand.

These responses offer a broad meaning for the word fluency and how it can be assessed that extends beyond basic facts and the traditional timed test. Teacher 2HC also commented that the mathematical reasoning aspect of fluency involves students “explaining why and how and not just to me, but to each other.”

Regarding addition and subtraction of numbers within 1000, 2HC provided a very succinct description of efficiency, “that it works, works well, and it’s quick.” Teacher 2HC did not appear to equate efficiency with any particular calculation method and explained that the plural use of the word method in the standard:

because there’s more than one way to solve problems and people see things differently. If you’re adding two two-digit numbers, kids may add ones first and then add the tens together. Or they may see those ones and be able to automatically carry the ten to the tens column. There’s many ways to break numbers apart and put them back together.

To teacher 2HC, generalizable methods are “the common ways and, perhaps, the uncommon ways” of solving problems. “I think that the way that we teach math now helps students understand number sense and basics and they can break numbers apart and put them together more easily than when we were just trained in the rote method.” Teacher 2HC’s interpretations of the language in the standard suggested that there are many ways for students to efficiently add and subtract numbers within 1000. These described classroom practices align with the high-constructivist mathematics beliefs indicted by an overall mean score of 4.21 as well as responses to specific survey items.

Teacher 2HC strongly agreed with the following survey statement:

Item 41: *Given appropriate materials, children can create meaningful procedures for computation.*

Teacher 2HC’s practices of encouraging students to break apart and put numbers back together to solve problems not only indicates the belief that such activities help

students develop number sense, but also reflects the belief that people see problems—and therefore solutions—differently. This encouragement, expressed in 2HC’s quantitative and qualitative data, is a highly constructivist, broad interpretation of the words *fluency*, *efficiency*, and the plural *methods* used in the standard.

Teacher 2HC: CCSSM Implementation.

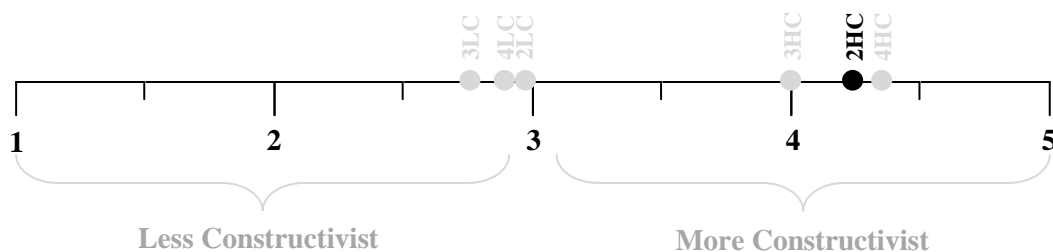
Beliefs about Mathematics. Figure 10 summarizes 2HC’s interview data related to this belief factor. Teacher 2HC identified prior knowledge of basic facts and conceptual understanding of how numbers can be broken apart and put together as important for understanding computational procedures such as regrouping. Teacher 2HC has students make an “exchange” (trade) to see the conservation of number value when adding and subtracting numbers within 1000. When relating both computation and conceptual understanding to word problems, 2HC feels word problems can not only be solved by students who are not yet proficient in their skills, but also serves to facilitate the learning of the those skills, stating:

They can and I’ve seen it with some of my students who struggle....if they are listening to the number story and then you let them work...they’ll draw pictures. They’ll figure out a way. I think number stories actually give kids more options than a simple addition or subtraction problem. It gives them the freedom to say, ‘oh, I can draw a picture. I can do an open number line’—which we do a lot—‘I can take out counters.’ They can do things with a number story that they might not necessarily do with

numbers that are just in columns with an addition or subtraction sign. So, I think it kind of opens doors.

Figure 10

Teacher 2HC: CCSSM Implementation and Beliefs about Mathematics



2HC Implementation Practices

- | | |
|--|--|
| <ul style="list-style-type: none"> • basic facts • place value understanding • understanding making a trade | <ul style="list-style-type: none"> • understanding numbers; number sense • addition and subtraction of zero through twenty and what that means • beyond basic facts • seeing how numbers work together; complementary numbers • place value understanding • understanding trading as an exchange • “They may understand place value. They may understand how numbers come together and are broken apart, but they may never be super fast at computation. But those kids, you know, are also the ones that surprise you. When there’s a number story with hard numbers, they can still figure it out.” • “They can and I’ve seen it with some of my students who struggle.” • “if they if they are listening to the number story and then you let them work...they’ll draw pictures. They’ll figure out a way.” • number stories give kids more options, more freedom to solve problems their own way • open number line; use counters • “They can do things with a number story that they might not necessarily do with numbers that are just in columns with an addition or subtraction sign. So, I think it kind of opens doors.” |
|--|--|

Clearly, these practices indicate that skills, understanding, and word problem solving should be integrated in the learning experience.

Teacher 2HC described implementation practices regarding the relationship among skills, understanding, and word problem solving that were highly constructivist in nature. This aligns with 2HC's overall mean score of 4.02, indicating beliefs that trend toward the highly constructivist end of the spectrum. Specific item analysis aligns with these described implementation practices regarding the relationship between skills and understanding and the sequencing of mathematics topics. Teacher 2HC strongly agreed with the following MBS statements:

Item 1: *Children should solve simple word problems before they master computational procedures.*

Item 6: *Children should understand the meaning of an operation (addition, subtraction, multiplication, or division) before they memorize number facts.*

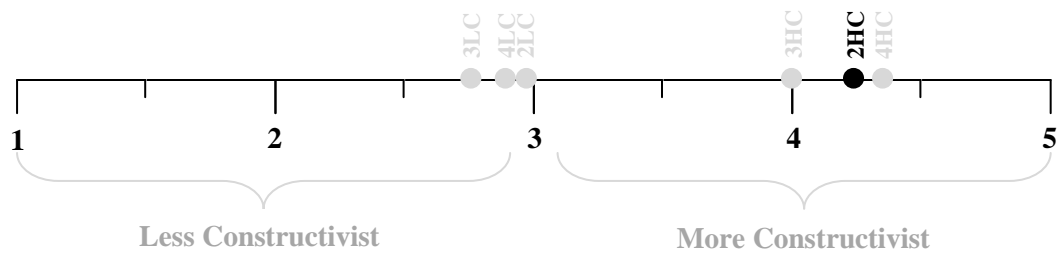
Item 25: *Children should understand computational procedures before they master them.*

Teacher 2HC demonstrated consistency within this belief factor, agreeing or strongly agreeing with constructivist statements and disagreeing with non-constructivist statements. This consistency in beliefs about the relationship between skills, understanding, and solving word problems is also reflected in 2HC's described practices used to implement the CCSSM standard.

Beliefs about Learning Mathematics. Figure 11 provides a summary of 2HC's interview data related to this belief factor.

Figure 11

Teacher 2HC: CCSSM Implementation and Beliefs about Learning Mathematics



2HC Implementation Practices

- | | |
|--|---|
| <ul style="list-style-type: none"> • paper-pencil (traditional) procedure | <ul style="list-style-type: none"> • draw a picture • expanded notation, partial sums • paper-pencil (traditional method) • strategies come from “them” (students) • hands-on in the beginning, • mathematical reasoning and explaining • Digi-Blocks, base-10 blocks • make numbers, show numbers (to get number sense) • place value—“so important...builds everything” • “we do all of that work so that when the procedure comes in, they understand what they’re doing and once they have that deeper understanding” • “every child is developmentally ready [for multi-step procedure] at a different time • breaking numbers apart, putting numbers together • teacher gives procedural support • discovery • students to learn strategies through facilitated, inquiry-based discussion and activities • “What works for you?” • students “need to figure out strategies on their own and develop them” • learning rote has no meaning; they need to build knowledge base |
|--|---|

2HC described several implementation practices relating to how children best learn mathematics that are constructivist in nature, aligning with 2HC's overall mean score of 4.21. Specific item analysis is consistent with these findings. Regarding how children best learn mathematics, 2HC strongly agreed with the following MBS statements:

Item 10: *Given appropriate materials, children can create meaningful procedures for computation.*

Item 44: *Allowing children to discuss their thinking helps them to make sense of mathematics.*

These beliefs influence this teacher's decision to implement the standard using a constructivist learning environment including manipulatives, games, and invented/multiple computation strategies. When asked from where these different strategies come, 2HC immediately responded: "them" (meaning the students) and explained, "I try to let the kids discover, and tell me, and then teach each other." This practice of letting students discover mathematics relationships for themselves requires them to play an active role in the classroom: they do not learn just by listening to the teacher, but learn from using materials, answering questions, and teaching one another.

This student-centered implementation model reflects 2HC's high-constructivist beliefs on the MBS. In fact, 46 of 48 (95.83) of 2HC's survey responses demonstrated agreement with constructivist (or disagreement with non-constructivist) statements. The strong consistency within belief statements and between beliefs and practices is perhaps best expressed in 2HC's own words:

They can learn rote, but it has no meaning. So, if they're discovering it on their own and their getting their hands on things and they're building and they're guessing and their using trial and error and coming up with strategies, it's just...they're so much more involved and they learn so much more and it's so much more meaningful to them and they hold onto that. And they're building their knowledge base and you just cannot replace that.

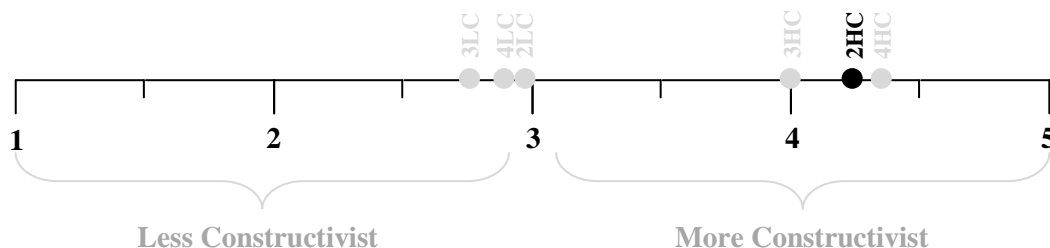
Beliefs about Teaching Mathematics. Figure 12 provides a summary of 2HC's interview data related to this belief factor. Teacher 2HC provided detailed descriptions of constructivist implementation practices that reflect the teaching-as-facilitator instructional model through which teachers "create the conditions that will allow students to take their own effective mathematical actions" (Smith, 1996, p. 393). Teacher 2HC encourages students to "teach each other," encouraging multiple voices in the mathematics classroom. The overwhelming majority of implementation practices reflect 2HC's constructivist beliefs about teaching that connect with his or her constructivist beliefs about mathematics and the ways students learn.

These beliefs are particularly indicated by 2HC's strong agreement with the following MBS statement:

Item 9: Mathematics should be presented to children in such a way that they can discover relationships for themselves.

Figure 12

Teacher 2HC: CCSSM Implementation and Beliefs about Teaching Mathematics



2HC Implementation Practices

- | | |
|---|---|
| <ul style="list-style-type: none"> • “I definitely teach them how” • “I actually teach partial sums backward[s] because I want them to always start with the ones.” | <ul style="list-style-type: none"> • “lead by discovery” • “hands-on math is so important at a young age...and ... even as the kids get bigger” • “if you facilitate and give them enough opportunities, they’ll get to the point where they are ready to take the leap” • students figure out own strategies and develop them • offer procedural, place value support • students teach each other • having students explain thinking and mathematical reasoning • “the way that we teach math now helps students understand number sense and basics and they can break numbers apart and put them together more easily than when we were just trained in the rote method.” • “I actually teach partial sums backward[s] because I want them to always start with the ones.” • strategies come from students • “we talk about what strategies they might use” • “let the kids discover, and tell me, and then teach each other” • “I let them go at their own pace.” • “each child learns differently and they have to do what works for them” • “provide them with opportunities, repeatedly, throughout the year... spiraling and growing with numbers as the numbers become bigger” • “they will always come up with ways to solve problems if you provide them the opportunity” |
|---|---|

During the interview, 2HC explicitly referenced New Math, stating “the way that we teach math now helps students understand number sense and basics and they can break numbers apart and put them together more easily than when we were just trained in the rote method.” Teacher 2HC’s constructivist implementation of this CCSSM standard clearly reflects a mathematics teacher who believes that students “will always come up with ways to solve problems if you provide them the opportunity.”

Comparative Analysis of 2LC and 2HC. The analysis between these two second grade teachers finds that mathematics beliefs influence teacher practices related to the CCSSM, from shaping how teachers interpret the language of the standard to the actions they take when working with students in the classroom. One critical example from the data illustrates how differences in mathematics beliefs influence two teachers to implement the same CCSSM in very different ways.

Both second grade teachers describe telling students to start with the ones when adding and subtracting numbers within 1000. On the surface, these responses appear very much the same; the researcher placed 2HC’s comment in both the less constructivist and more constructivist categories because it is open to interpretation. However, this researcher used the teachers’ elaborated responses to reveal differences in both beliefs and practice. Teacher 2LC tells students to start “at the right, with the ones” while using a place value or t-chart. This teach-by-telling is modeled (via direct instruction) using one strategy and one method for the computation. Teacher 2HC, however, allows students to create their own way of putting numbers together and taking them apart when adding and subtracting with 1000, stating “kids may add ones first and then add the tens together.”

Teacher 2HC tells students using the partial sums method to add the ones first. This comment in isolation seems to reflect the traditional teach-by-telling instructional model as 2LC. However, when placed into context by a later response, 2HC reasoned that organizing the sums in this manner helped students develop a better conceptual understanding to transition to the standard computational procedure, thus bridging the gap between the alternate and traditional algorithm that “eventually clicks in when that child is developmentally ready.”

The following MBS statement generated very different responses from 2LC and 2HC:

Item 48: *Children learn mathematics best from teachers’ demonstrations and explanations.*

Teacher 2LC agreed with statement and the implementation of the standard, using an anchor chart to model the traditional computation method, reflects this belief. By contrast, 2HC strongly disagreed with this statement and implemented the standard using a variety of student-generated computation strategies. Thus, differences in beliefs directly influence differences in interpretation and implementation of the same Grade 2 CCSSM mathematics standard.

Interviews supported the quantitative finding that these teachers’ beliefs differed most often regarding the teacher’s role in the mathematics room. One distinct finding, that supports the researcher’s decision to use a mixed methods study design, is that teaching practices may reveal differences in the ways teachers interpret a survey item. While both teachers agreed (2LC) or strongly agreed (2HC) with the notion (Item 9) that

mathematics should be presented so that children discover mathematical relationships for themselves, the interviews revealed that 2LC's practices align with the first portion (presentation) of the statement while 2HC's practices align with the second (discovery) portion of the statement.

Individual Teacher Profiles: Grade 3. In the Grade 3 CCSSM standard selected for this study, students “develop an understanding of the meanings of multiplication and division of whole numbers through activities and problems...using increasingly sophisticated strategies” (Introduction, p. 21) to “fluently multiply and divide within 100” (Standard 3.OA7, p. 23). The first two interview questions encouraged teachers to explain the meaning of the word *fluency*, the phrase *increasingly sophisticated strategies*, and the plural use of the word *strategies* in the standard.

The remaining interview questions asked teachers to describe implementation practices related to each of the three belief factors in the study: Mathematics (the relationship among skills, understanding, and word problem solving; sequencing of mathematics topics), Learning Mathematics, and Teaching Mathematics. Two questions (related to beliefs about mathematics) asked what prior skills and understanding students need to multiply and divide within 100 and how skills (basic facts and computational procedures), conceptual understanding of both operations, and word problem solving are related. Sub-questions specifically asked whether students should master skills before word problem solving. Two questions (related to beliefs about learning mathematics) asked teachers to describe the strategies students use to multiply and divide within 100, how students learn those strategies, and how students demonstrate their learning of the

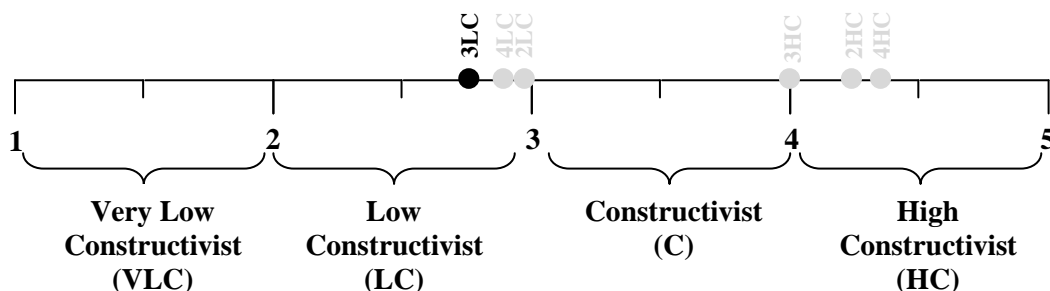
standard. The final interview question (related to beliefs about teaching mathematics) asked teachers to describe their role in helping students meet the standard. (See Appendix C for complete sets of Grades 2, 3, and 4 interview questions.)

Results for each of the third grade teachers include a teacher profile, interpretation results, and implementation results organized by belief factor. The third grade section ends with a brief comparative analysis of teachers 3LC and 3HC.

Grade 3 Low Constructivist (Teacher 3LC) Profile. Teacher 3LC teaches third grade students at elementary school 3 and has one year of teaching experience. Teacher 3LC earned an overall mean score of 2.73 on the survey, placing the beliefs as low-constructivist on the continuum as shown in Figure 13. Although this score is near the neutral score of 3, teacher 3LC earned the lowest score of any teacher (n=80) who took the survey.

Figure 13

Teacher 3LC: Placement on Constructivist Continuum



Teacher 3LC: CCSSM Interpretation. Teacher 3LC stated “Fluency to me means that they’re accurately being able to master a specific skill within a time frame—a given time frame—or by just basic showing of the skill through an academic worksheet or an

oral response.” Teacher 3LC then described a program used to help students memorize basic facts by setting goals for a one-minute timeframe. Completing the program is a way for students to demonstrate fluency in this teacher’s classroom. “So fluency to me is that they can master it or say it out, without hesitation. I think if a student is able to when they hear it they can say it so that, you know, it’s almost like it’s a not-think, it’s automated for them would mean fluency to me.”

For teacher 3LC, the words *increasingly sophisticated strategies* involves “very complex strategies after building on the very simple strategies...that will almost enhance their ability to solve some type of multiplication or division problems...a higher level thinking strategy.” Teacher 3LC also suggested that increasingly sophisticated strategies involved students’ developmental movement from needing to see the problem visually (by drawing a picture, drawing an array, or making equal groups) to solving the problem arithmetically. Teacher 3LC interpreted the plural use of the word *strategies* to mean that different students use different strategies and that even the same student may use “one or two or a handful of different strategies.”

Teacher 3LC’s interpretation of the language used in the standard reflects both constructivist and traditional beliefs. The researcher used specific item analysis to determine whether these interpretations aligned with certain beliefs on the MBS. Emphasis on basic fact timed tests and a program to practice these skills reflected 3LC’s agreement with the following MBS statement:

Item 35: *Frequent drills on the basic facts are essential in order for students to learn them.*

However, 3LC also agreed with this MBS statements:

Item 6: *Children should understand the meaning of an operation (addition, subtraction, multiplication, or division) before they memorize number facts.*

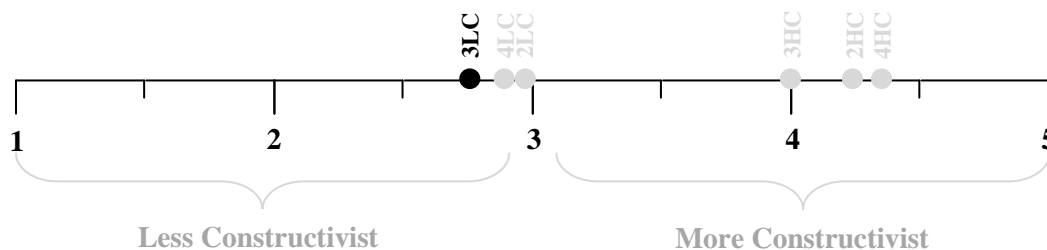
In a later response, 3LC identified the importance of helping students develop a conceptual understanding of multiplication as repeated addition and division as repeated subtraction. This teacher also emphasized a student's ability to visualize equal groups using drawings or an array, as indicated in the standard. These practices, relating to fluency with and strategies for multiplication and division, are constructivist in nature. Teacher 3LC's emphasis on isolated basic fact drills is, by itself, a more traditional approach. However, when combined with strategies that encourage a conceptual understanding of these operations, the teacher described a balanced interpretation aligned with beliefs that trend toward neutral.

Teacher 3LC: CCSSM Implementation.

Beliefs about Mathematics. Figure 14 summarizes 3LC's interview data related to this belief factor. Teacher 3LC identified both basic fact knowledge as well as a conceptual understanding of multiplication and division as operations to meet the standard: "So, definitely addition and subtraction, the concepts of repeated addition, repeated subtraction is huge... a big skill for them to be able to understand that...and I think visually, they need to understand the concept of what that would look like. What would seven groups of three look like?" Teacher 3LC's practice of using several strategies to help students connect basic fact skills and conceptual understanding reflects

Figure 14

Teacher 3LC: CCSSM Implementation and Beliefs about Mathematics



3LC Implementation Practices

- | | |
|---|--|
| <ul style="list-style-type: none"> • basic facts mastery through timed tests • program to practice basic facts in isolation • students who are not visual learners need to master basic facts before problem solving • not knowing facts leads to frustration, block understanding of problem • “skills of multiplication and division should foremost be mastered off the bat before we really dig deep into a word problem” • “I would never teach word problems first, before I teach multiplication and division facts • “I would definitely want those facts mastered before I start really digging into the nitty-gritty of a multi-step word problem • mastery of facts will effect success in solving word problems | <ul style="list-style-type: none"> • understanding of multiplication as repeated addition, division as repeated subtraction • subtraction mastered before division • mastery of addition and subtraction • visual understanding: “What would seven groups of three look like?” |
|---|--|

constructivist beliefs about the importance of integrating skills and understanding, as indicated by agreement with the following MBS statement:

Item 25: Children should understand computational procedures before they master them.

Teacher 3LC, however, showed inconsistencies between beliefs. Agreement with Item 25, responses to other related survey items, and 3LC's description of implementation practices in the classroom directly contradict the teacher's disagreement with this MBS statement:

Item 22: Recall of number facts should precede the development of an understanding of the related operation.

While some of 3LC's expressed beliefs and many classroom practices were constructivist in nature, once the mathematics relationship included word problem solving, 3LC took a decidedly traditional turn. Teacher 3LC stated "I would never teach word problems first, before I teach multiplication and division facts" based on the view that basic fact mastery will prevent frustration in solving problems, especially for students who are not visual learners. Lack of these skills may "block their understanding of the rest of the problem." This traditional view of facts first was confirmed by numerous survey responses, including disagreement with these MBS statements:

Item 4: Time should be spent solving simple word problems before children spend much time practicing computational procedures.

Item 13: Children should have many informal experiences solving simple word problems before they are expected to memorize number facts.

Teacher 3LC's implementation of the standard and beliefs about relationships among skills, understanding, and word problem solving are simultaneously traditional and constructivist in nature. Teacher 3LC's overall mean score of 2.73 appears to reflect both constructivist beliefs about relating skills (basic fact and computational procedures)

and conceptual understanding of operations and the more traditional beliefs that skills and procedures must be mastered before solving word problems, influencing a slightly less constructivist implementation of the standard.

Beliefs about Learning Mathematics. Figure 15 provides summary of 3LC's interview data related to this belief factor. Teacher 3LC described both traditional and constructivist implementation practices related to how children best learn mathematics. This aligns with 3LC's overall mean score of 2.73, indicating beliefs that trend toward a neutral score of 3. Students in 3LC's classroom use multiple strategies to multiply and divide numbers within 100. This teacher emphasized the importance of meeting students' needs and encouraging them to use multiple strategies "because not all these kids learn the same way."

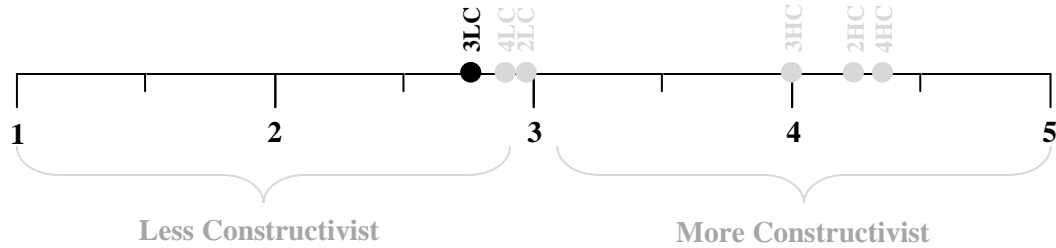
Specific item analysis found these constructivist implementation practices are consistent with some of 3LC's responses relating to the learning mathematics belief factor, including agreement with this MBS statement:

Item 36: Most children can figure out a way to solve many mathematics problems without any adult help.

However, 3LC also expressed contradictory beliefs and practices. Three times during the interview, 3LC commented on the importance of helping students avoid struggle and frustration. This is inconsistent with a constructivist perspective that recognizes disequilibrium (Fosnot, 2005) and struggle (Stigler in Spiegel, 2012) as a natural part of

Figure 15

Teacher 3LC: CCSSM Implementation and Beliefs about Learning Mathematics



3LC Implementation Practices

<ul style="list-style-type: none"> • basic fact practice • strategies “that I will give them” • lots of rigorous practice • worksheets • timed tests • use of models indicates lower level of understanding • learning should occur in stages for non-visual learners: facts before word problem solving • frustration should be avoided when possible • learning without struggle 	<ul style="list-style-type: none"> • repeated addition for multiplication • repeated subtraction for division • visual representation of multiplication: pictures • arrays, area models • arithmetic • different strategies (different students and one student) • handful of multiplication strategies • making equal groups • songs, rhymes, games, tricks • multiple strategies • some students come up with strategies on their own • quizzes and assessments • observations of “how the interaction is between student to student” • “not all kids learn the same way”
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the learning process. Specific item analysis suggested 3LC also holds traditional beliefs about mathematics learning, indicated by strong agreement with the following MBS statement:

Item 45: *To be successful in mathematics, a child must be a good listener.*

This statement appears to suggest students learn by listening to the teacher, but does not explicitly state that position. The responder may have interpreted this statement to

include the act of students listening to each other and, therefore, it is difficult to accurately interpret the intention of the responder. Other statements however are less vague in meaning. 3LC agreed with this MBS statement:

Item 26: *Children learn math best by attending to the teacher's explanations.*

Teacher 3LC also disagreed with the following statement:

Item 33: *Children can figure out ways to solve many math problems without formal instruction.*

The indication of both constructivist and non-constructivist beliefs on the MBS, resulting in an overall mean score of 2.73 influences a teacher to implement a mathematics standard in ways that are both traditional and constructivist.

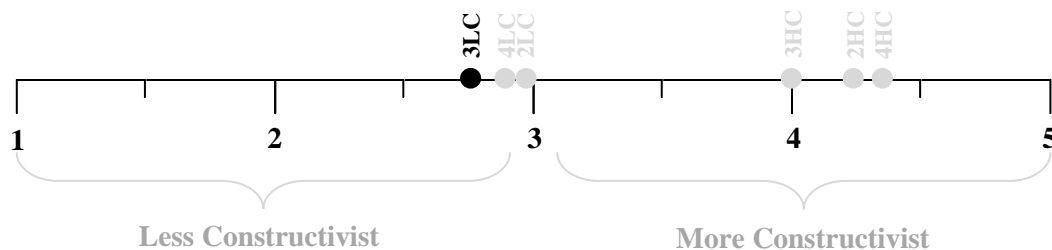
Beliefs about Teaching Mathematics. Figure 16 provides a summary of 3LC's interview data related to this belief factor. Teacher 3LC described some implementation practices that were constructivist in nature and others that reflected the traditional model of teaching-as-telling, consistent with 3LC's overall mean score of 2.73 which trends toward neutral. Specific item analysis indicated several contradictory beliefs about mathematics teaching. For example, 3LC disagreed with following MBS statements:

Item 2: *Teachers should encourage children to find their own solutions to math problems even if they are inefficient.*

Item 37: *Teachers should allow children to figure out their own ways to solve simple word problems.*

The teaching-as-telling model and avoidance of struggle is further confirmed by 3LC's agreement with this MBS statement:

Figure 16

Teacher 3LC: CCSSM Implementation and Beliefs about Teaching Mathematics**3LC Implementation Practices**

- | | |
|---|---|
| <ul style="list-style-type: none"> • emphasis on timed tests for basic facts • basic fact drill • strategies “that I will give them” • interprets modeling as lower level thinking • “I would never teach word problems first, before I teach multiplication and division facts.” • rigorous practice with facts • teaching to avoid learning struggle • mastery of skill affects ability to problem solve • “I’m willing to listen and hear what it would look like to them.” | <ul style="list-style-type: none"> • helping students use models to develop conceptual understanding • need for multiple strategies because all students are different • assessment through observation of student-student interaction during games • using student thinking/performance to guide daily math message • “I’m willing to listen and hear what it would look like to them.” • “to meet all of their learning needs is really just to hit that standard but not be so specific in one way of teaching... especially when it comes to math, because not all these kids learn the same way” |
|---|---|

Item 7: The teacher should demonstrate how to solve simple word problems before children are allowed to solve word problems.

Teacher 3LC’s interview responses indicate willingness to try a variety of instructional methods: “To meet all of their learning needs is really just to hit that standard but not be so specific in one way of teaching... especially when it comes to math, because not all these kids learn the same way.” Teacher 3LC stated “I’m willing to listen and hear what it would look like to them” and acknowledges the creativity of students. The researcher

categorized this comment as both less constructivist and more constructivist; while 3LC is open to the idea of listening to students express their thinking, this response does not indicate that this teacher uses students' mathematical thinking as a major factor to guide instruction. Teacher 3LC's survey responses (indicating beliefs) and described implementation practices appear to reflect either a contradictory or neutral perspective on how multiplication and division should be taught. Even the following MBS statement, pertaining to instructional goals, received a neutral response from 3LC:

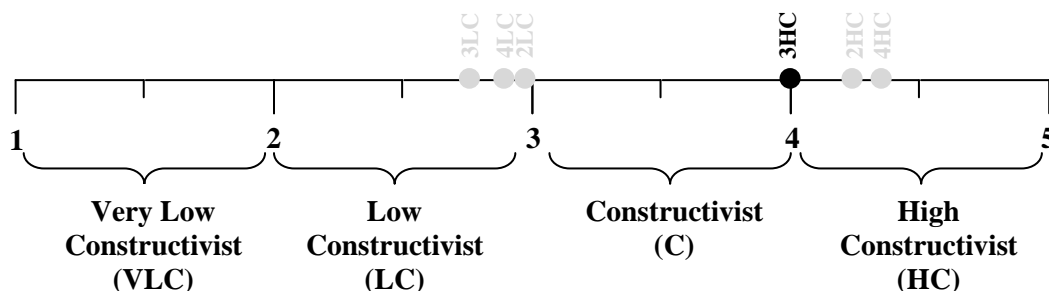
Item 30: *The goals of instruction in mathematics are best achieved when students find their own methods for solving problems.*

These inconsistencies may be the result of inexperience. Teacher 3LC is a rookie who perhaps has come into the classroom with traditional views of teaching efficacy as described by Smith (1996) in which students learn from the teacher's demonstrations and explanations. This internal professional conflict between traditional and constructivist practices may be reflected in 3LC's adoption of many, and sometimes conflicting, instructional approaches. Teaching with the CCSSM, however, involves modeling, reasoning, and multiple ways of solving problems—practices 3LC has clearly implemented. Over time, such practices may effect a change in 3LC's beliefs about how mathematics should be taught.

Grade 3 High Constructivist (Teacher 3HC) Profile. Teacher 3HC teaches third grade students at elementary school 3 and has 16 years of teaching experience. Teacher 3HC earned an overall mean score of 4.02 on the MBS survey, as shown in Figure 17, suggesting this teacher holds high-constructivist beliefs.

Figure 17

Teacher 3HC: Placement on Constructivist Continuum



Teacher 3HC: CCSSM Interpretation. Teacher 3HC stated fluency is “the ability to recall a fact quickly and accurately without having to think too much about it. It’s an automaticity sort of thing.” This teacher explained how students demonstrate fluency through a program in which “the children read, write, and listen to facts and they’re quizzed for a minute and a half daily” and on daily math work in their “ability to quickly and accurately come up with an answer.”

For teacher 3HC, the words *increasingly sophisticated strategies* means moving from less sophisticated strategies, for example using “quick addition” for a multiplication problem, to also “being able to use a variety of strategies and to show those strategies with pictures, with explanations” which goes beyond rote memory. Teacher 3HC interpreted the plural use of the word *strategies* to mean that “there’s more than one way” to solve a mathematics problem. Two similes explain this concept to students:

When I talk about strategies with the students, I tell them it’s like a path to a destination and there’s more than one path to get to that destination. I tell them it’s kind of like eating a Reese’s Peanutbutter Cup; there’s more than

one way to eat a Reese's Peanutbutter Cup. Your way might be different than my way, but it's your way of getting to an accurate answer...

Teacher 3HC's interpretation of the language used in the standard reflects constructivist beliefs. The researcher used specific item analysis to determine whether these interpretations aligned with certain beliefs on the MBS. 3HC strongly agreed with the following MBS statements:

Item 35: Frequent drills on the basic facts are essential in order for students to learn them.

However, this was the only non-constructivist item on the entire 48-item MBS with which 3HC strongly agreed. However, 3HC also agreed with this MBS statement:

Item 6: Children should understand the meaning of an operation (addition, subtraction, multiplication, or division) before they memorize number facts.

This belief aligns with the use of the basic fact program as one of several classroom activities to help students develop fluency and understanding of these operations. Teacher 3HC also mentioned the ability to explain multiplication, show it with pictures, and use in games and in daily work solving problems, reflecting a more constructivist interpretation that aligns with an overall mean score of 4.02.

Teacher 3HC also indicated the importance of helping students develop an initial conceptual understanding of multiplication as repeated addition by showing the operation in pictures and explanations. The program used to practice basic facts was not isolated from daily work with problems, again reflecting the constructivist beliefs indicated by 3HC's overall mean score.

Teacher 3HC: CCSSM Implementation.

Beliefs about Mathematics. Figure 18 provides a summary of 3HC's interview data related to this belief factor. Teacher 3HC states that students should have number sense, fluency with math facts, the ability to look for a smaller problem, and an understanding of numbers that enables them to determine whether the answer “really makes sense.” This response indicates both prior basic fact skills and conceptual understanding of multiplication and division are needed to meet the standard. Teacher 3HC's practice of encouraging students to connect basic fact skills with a conceptual understanding of multiplication and division reflects constructivist beliefs about the importance of integrating skills and understanding, as indicated by strong agreement with the following MBS statement:

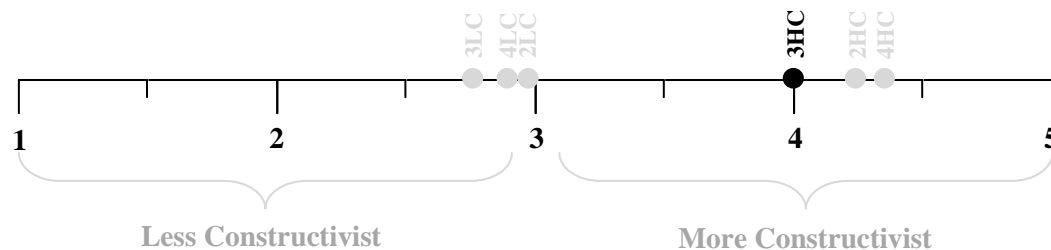
Item 25: *Children should understand computational procedures before they master them.*

Within this belief factor, 3HC consistently agreed or strongly agreed with constructivist statements and disagreed with non-constructivist MBS statements including:

Item 22: *Recall of number facts should precede the development of an understanding of the related operation.*

The use of constructivist implementation extended to practices that integrate skills, understanding, and solving word problems. Teacher 3HC finds that solving word problems can motivate students to learn their basic facts. Regarding the sequences of skills and word problems, 3HC compares it to “the chicken or the egg—I don't think one has to come before the other.” When asked about exposing students who have yet to

Figure 18

Teacher 3HC: CCSSM Implementation and Beliefs about Mathematics**3HC Implementation Practices**

- | | |
|--|---|
| <ul style="list-style-type: none"> • fluency with math facts • program to practice basic facts | <ul style="list-style-type: none"> • number sense • understanding of math facts • know how to look for smaller, related problems • extended facts • need to understand numbers to know if an answer really makes sense • “There’s a lot to be said for learning the concept of multiplication and working through the facts— that ‘wow, that took me a really long time to draw an array for eight times fives. I wonder if there’s a quicker way to get to that answer?’” • word problems as motivation to know facts • “I don’t think one has to be done before the other. I think maybe you get a better understanding if you do it all together.” • “the chicken or the egg—I don’t think one has to come before the other.” • word problems before facts: “That’s what we do. I mean, everybody kinds of works at their own pace memorizing their facts and we can’t wait until everyone learns them to expose them to that. And even if they haven’t memorized them, they do have strategies and tools in their little tool kit to come to find out what eight times five is.” • calculators • multiplication charts in journals encouraged • area models, number line models • skip counting |
|--|---|

master their basic facts, 3HC replied, “That’s what we do. I mean, everybody kinds of works at their own pace memorizing their facts and we can’t wait until everyone learns them to expose them to that. And even if they haven’t memorized them, they do have strategies and tools in their little tool kit to come to find out what eight times five is.” These ways of implementing the third grade CCSSM multiplication and division within 100 standard reflects 3HC’s responses to specific items on the survey, including strong agreement with the following statements:

Item 12: *Most young children can figure out a way to solve simple word problems.*

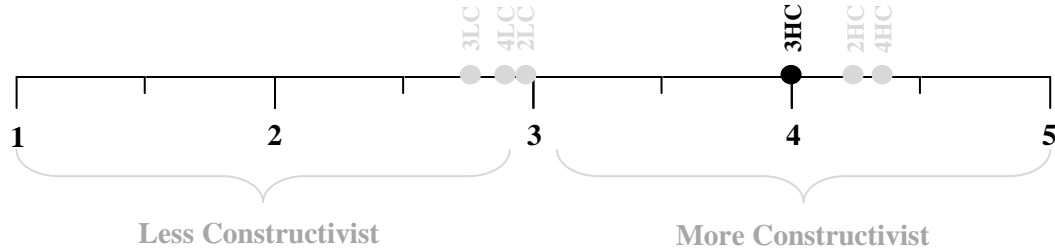
Item 13: *Children should have many informal experiences solving simple word problems before they are expected to memorize number facts.*

Teacher 2HC’s constructivist implementation practices and beliefs about mathematics—reflecting the integration of skills, understanding, and word problem solving—are best expressed in the teacher’s own words: “I don’t think one has to be done before the other. I think maybe you get a better understanding if you do it all together.”

Beliefs about Learning Mathematics. Figure 19 summarizes 3HC’s interview data related to this belief factor. Teacher 3HC described a variety of constructivist learning activities used to implement this mathematics standard. These practices align with 3HC’s overall mean score of 4.02, indicating high-constructivist beliefs. Students in 3HC’s classroom use multiple strategies to multiply and divide numbers within 100. Throughout the interview, 3HC emphasized there are several ways to solve mathematics problems.

Figure 19

Teacher 3HC: CCSSM Implementation and Beliefs about Learning Mathematics



3HC Implementation Practices

- | | |
|---|--|
| <ul style="list-style-type: none"> • basic fact practice • “the best way is to just memorize” • “We do daily math practice to help them memorize their facts.” | <ul style="list-style-type: none"> • fingers, tallies • pictures • neighbor facts • elevens twins • relating fives tables to clock minute hand • counters • arrays • area models • multiplication as repeated addition • strategies come from progress through years, understanding of base-10 system • “experience in the world just helps them understand everything...on the playground and sharing cookies, and all those life experiences they bring into classroom that they can relate to [the standard]” • assess understanding through games • informal and formal assessment, written assessments • “they’re all kind of at their own level and I monitor where they are” • “... explaining how they got their answer. They bring up their work to the document camera and they share and explain it to their peers.” |
|---|--|

Specific item analysis found these implementation practices are consistent with 3HC’s responses relating to this belief factor, including agreement with this MBS statement:

Item 36: *Most children can figure out a way to solve many mathematics problems without any adult help.*

Teacher 3HC also expressed the notion that children naturally learn mathematics outside the classroom and that “their experience in the world just helps them understand everything...on the playground and sharing cookies, and all those life experiences they bring into classroom that they can relate to [the standard]”. This comment reflects the belief that the teacher is one of many sources of mathematical knowledge, as indicated with 3HC’s strong agreement with this MBS statement:

Item 33: *Children can figure out ways to solve many math problems without formal instruction.*

Similarly, 3HC encourages students to have a voice in the mathematics classroom by “explaining how they got their answer. They bring up their work to the document camera and they share and explain it to their peers.” This practice enables students to learn with and from each other and is consistent with 3HC’s strong agreement with this related MBS statement:

Item 31: *Allowing children to discuss their thinking helps them to make sense of mathematics.*

Finally, 3HC exhibits implementation practices consistent with constructivist beliefs about learning by allowing students to solve problems in their own way and evaluate their choice of strategy. “There’s a lot to be said for learning the concept of multiplication and working through the facts—that ‘wow, that took me a really long time to draw an array for eight times fives. I wonder if there’s a quicker way to get to that answer.’” This

metacognition encourages students to learn how to learn. Learning a concept through work, the ability to “barrel through it as best you can” suggests that struggle is a natural part of the learning process. This constructivist belief is consistent with 3HC’s overall mean score, aligns with other implementation practices, and appears to guide instruction.

Beliefs about Teaching Mathematics. Figure 20 summarizes 3HC’s interview data related to this belief factor. Throughout the interview, 3HC described constructivist teaching practices, consistent with an overall mean score of 4.02, indicating high-constructivist beliefs as measured in the MBS survey. Teacher 3HC exhibits the teacher-as-facilitator model of efficacy in which teachers “create the conditions for students to be active learners (Smith, 1996). The teacher provides students with “all different kinds of opportunities to learn” about multiplication and division. These described implementation practices appear to be influenced by constructivist beliefs about mathematics teaching, as indicated by 3HC’s strong agreement with the following three MBS statements:

Item 9: *Mathematics should be presented to children in such a way that they can discover relationships for themselves.*

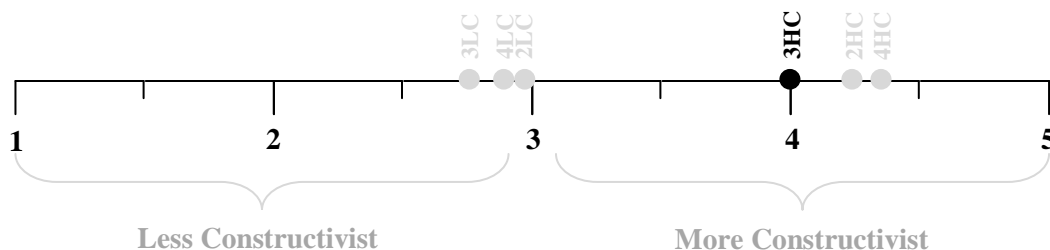
Item 37: *Teachers should allow children to figure out their own ways to solve simple word problems.*

Item 30: *The goals of instruction in mathematics are best achieved when students find their own methods for solving problems.*

Teacher 3HC’s responses on the MBS were consistently constructivist, 45 of 48 items (93.75%) were neutral or in agreement with constructive mathematics beliefs. This teacher expressed the importance of basic facts, which is typically associated with a more

Figure 20

Teacher 3HC: CCSSM Implementation and Beliefs about Teaching Mathematics



3HC Implementation Practices

<ul style="list-style-type: none"> • time to practice rote memorization • equating basic facts with phonics 	<ul style="list-style-type: none"> • helping students develop really good numbers sense • “building on all those skills that they have” • combination of giving time to practice rote memorization” so they can focus on the meaning of the problem • comparing knowing basic facts to knowing letter sounds and blends in reading • exposing them to all different kinds of problems • “I try to give them the tools that I can to be successful and then just the experiences with these numbers and these kinds of problems.” • “Good teaching is a mixture. Having them try out a problem independently. I do a lot of peer teaching where they work with a partner. Certain things, I just model outright....all different kinds of opportunities to learn. • “Your way might be different than my way, but it’s your way of getting to an accurate answer...” • relating mathematics to real world • sharing own experiences learning mathematics
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traditional model. However, integrating skill practice with real-world connections and word problems was very important. Teacher 3HC also helps students see the similarities

between basic facts and phonics—knowing the facts allows one to focus on deeper meaning. Teacher 3HC describes his or her role as “helping students develop really good numbers sense” by “building on all those skills that [students] have” and trying to “give them the tools that I can to be successful and then just the experiences with these numbers and these kinds of problems.”

Comparative Analysis of 3LC and 3HC. The analysis between these two third grade teachers finds that mathematics beliefs influence teacher practices related to the CCSSM, from shaping how teachers interpret the language of the standard to the actions they take when working with students in the classroom. One critical example from the data illustrates how differences in mathematics beliefs influence two teachers to implement the same CCSSM in very different ways.

Both teachers 3LC and 3HC use a variety of strategies to help students learn to multiply and divide within 100 based on the belief that students learn differently and there many ways to solve mathematics problems. However, their differences in beliefs about mathematics (the relationship among skills, understanding, and word problem solving; the sequencing of mathematics topics) influence them to adopt very different implementation practices in the classroom. Teacher 3LC places such a strong emphasis on mastering basic facts, “I would never teach word problems first, before I teach multiplication and division facts” despite observing students successfully solve word problems using strategies (such as repeated addition), manipulatives, or conceptual models (pictures showing equal groups). By contrast, while teacher 3HC acknowledges that fact fluency makes solving word problems easier, he or she finds “there’s a lot to be

said for learning the concept of multiplication and working through the facts—that ‘wow, that took me a really long to draw an array for eight times fives. I wonder if there’s a quicker way to get to that answer.’...So I don’t think one has to be done before the other. I think maybe you get a better understanding if you do it all together....the chicken and the egg—I don’t think one has to come before the other.” For 3HC word problems appear to be not an *end*, but rather a *means* to skill mastery; thus integrating skills, understanding, and word problems to make learning more meaningful. For teacher 3LC, it’s a facts-first approach.

The following MBS statement generated very different responses from 3LC and 3HC:

Item 10: *Even children who have not learned basic facts can have effective methods for solving problems.*

Teacher 3HC strongly agreed with this statement while 3LC disagreed. These differences in beliefs clearly influence teachers to implement the same Grade 3 CCSSM mathematics standard in different ways.

Individual Teacher Profiles: Grade 4. The Grade 4 CCSSM standard selected for this study involves multi-digit multiplication using strategies based on place value and properties of operations, including the ability to explain calculation procedures (NGA & CCSSO, 2010, Number and Operations in Base Ten Standard 5, p. 29). The first two interview questions encouraged teachers to explain the meaning of the word “fluency” and the phrase “efficient procedures for multiplying whole numbers” in the CCSSM Grade 4 Introduction section (p. 27).

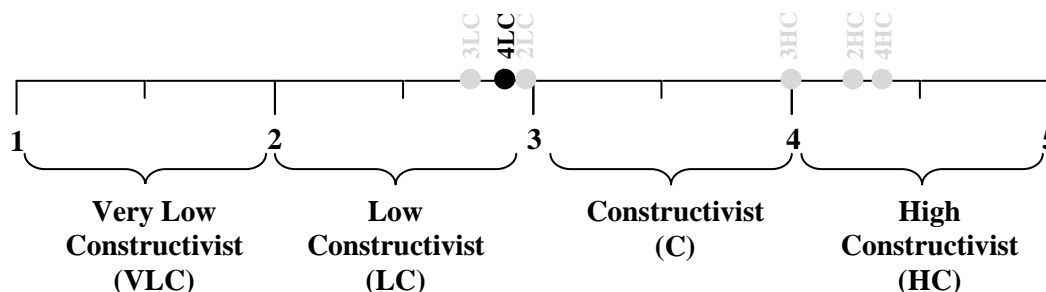
The remaining interview questions asked teachers to describe implementation practices related to each of the three belief factors in the study: Mathematics (the relationship among skills, understanding, and word problem solving; sequencing of mathematics topics), Learning Mathematics, and Teaching Mathematics. Two questions (related to beliefs about mathematics) asked what prior skills and understanding students need to multiply multi-digit numbers and how skills (basic facts and computational procedures), conceptual understanding of multiplication, and word problem solving are related. Sub-questions specifically asked whether students should master skills before word problem solving. Two questions (related to beliefs about learning mathematics) asked teachers to describe the multiplication strategies students use, how students learn those strategies, and how students demonstrate their learning of the standard. The final interview question (related to beliefs about teaching mathematics) asked teachers to describe their role in helping students meet the standard. (See Appendix C for complete sets of Grades 2, 3, and 4 interview questions.)

Results for each of the fourth grade teachers include a teacher profile, interpretation results, and implementation results organized by belief factor. The fourth grade section ends with a brief comparative analysis of teachers 4LC and 4HC.

Grade 4 Low Constructivist (Teacher 4LC) Profile. Teacher 4LC works with fourth graders at elementary school 2 and has 16 years experience teaching fourth and second grade students. Teacher 4LC earned an overall mean score of 2.88 on the survey, suggesting this teacher holds low-constructivist beliefs (as shown in Figure 21), but with a numerical score close to a neutral score of 3.

Figure 21

Teacher 4LC: Placement on Constructivist Continuum



Teacher 4LC: CCSSM Interpretation. To open the interview, 4LC explained “fluency to me would be that the student understands the material, can correctly interpret, like if there’s a question, what the question is asking. The student is able to understand the question so they can put their thoughts and their mathematical reasoning on paper.” In relation to multiplication facts, 4LC continues, “Fluency is that they know that fact and they can come up with the answer to that fact within a couple seconds.” Teacher 4LC then emphasized the importance of basic fact fluency, stating, “I do a lot of practice daily on math facts. I feel that that’s very important. It’s one of the most important things that a student should know are their facts.” The teacher described daily timed tests that vary according to a student’s ability. In 4LC’s classroom, students progress through the tables, within a shorter and shorter amount of time, beginning with lower facts first. Teacher 4LC is proud of the students’ historical success using this approach: “going from past years, all of the students in my class are above ninety-five percent in all operations, mathematical operations—we’re talking addition, subtraction, multiplication, and division.”

Regarding the meaning of efficient procedures in the language of the standard, 4LC explains “when I’m teaching the students how to do two-digit by two-digit multiplication...I go over a couple different algorithms for them to be successful. The majority of the time, to be efficient, students will usually choose the procedure that requires them to use the least amount of steps or is the least complicated.” Then teacher 4LC stressed the use of the traditional method of multiplication, stating “The majority of the time that is the traditional method of multiplication and that is more of what I tend to teach in the classroom because it also, when they move on to fifth grade, primarily that is the [algorithm] that they’re going to be using.” As with basic facts, 4LC described success teaching the traditional method of multiplication to students of all abilities.

Teacher 4LC mentioned three primary strategies that students use for multi-digit multiplication. “If we’re talking about multiplication, it would be partial products. There’s the lattice method...which I tend to not promote as much because it’s not looked upon... you know. In fifth grade, they should know the traditional method.” Teacher 4LC stressed the importance of students knowing the standard algorithm to be prepared for the next grade level and interpreted the meaning of *methods* in the standard as meaning the procedural steps of traditional multiplication or other methods.

These responses, particularly the emphasis on basic facts (and the daily use of timed tests) along with the clear preference for the traditional method of multiplication—though teaching partial products as well—aligns with 4LC’s overall mean score of 2.88 on the MBS, indicating beliefs that are low-constructivist in nature. These implementation practices suggest that 4LC equates efficiency with *a* particular method, in

this case the traditional method (or standard algorithm) of calculation, which is consistent with responses to specific survey items.

Teacher 4LC strongly agreed with the following MBS statement:

Item 35: *Frequent drills on the basic facts are essential in order for children to learn them.*

Teacher 4LC disagreed with this MBS statement:

Item 2: *Even children who have not learned basic facts can have effective methods for solving problems.*

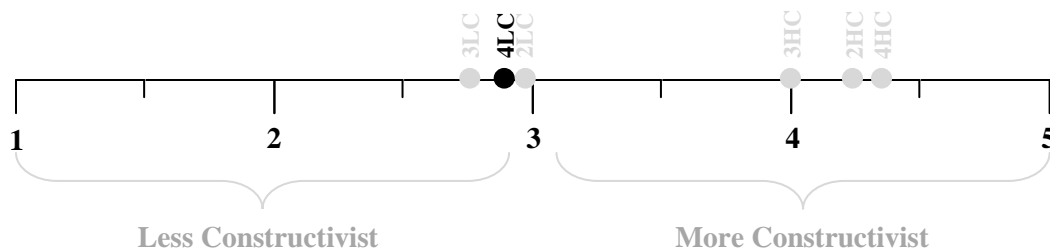
Teacher 4LC's practices of teaching students to use one particular (traditional) method for multi-digit multiplication aligns with the less-constructivist beliefs indicated on the survey, reflecting a narrow interpretation for fluency and what constitutes an efficient calculation method.

Teacher 4LC: CCSSM Implementation.

Beliefs about Mathematics. Figure 22 provides a summary of 4LC's interview data related to this belief factor. Teacher 4LC's responses make it clear that prior knowledge of basic facts is essential for solving multi-digit multiplication problems: "They need to know their math facts. They cannot be successful unless they know their math facts." A few comments, however, align with more constructivist beliefs: allowing students to struggle through a problem using other procedures or manipulatives to help them understand, letting students see for themselves (metacognition) that some methods are not as efficient, and building on prior knowledge to learn new concepts. Even so, most of these responses also contained the suggestion of a traditional perspective as well.

Figure 22

Teacher 4LC: CCSSM Implementation and Beliefs about Mathematics



4LC Implementation Practices

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| <ul style="list-style-type: none"> • “They need to know their math facts. They cannot be successful unless they know their math facts. And that’s also, when they’re sitting there, you almost get, at times, they feel like a peer pressure unless they know it.” • “They want to be successful. I’ve taken students that have received support in mathematics and they’ve been very successful once they knew their math facts.” • “breaking it into parts” based on basic facts • “breaking it down... if we’re multiplying in the ones column, I have them break it down into what they’re familiar with and what their secure with. Most of the time they’re secure with their twos, their fives, so we use that as a base for them to move on to the next set of numbers.” • “understand it faster” | <ul style="list-style-type: none"> • “I always relate word problems to real life.” • “everything’s practical and I want them to understand what is the purpose to solving this problem” • “I think they can solve it, but they definitely see the benefits of improving their facts and mastering their facts.” • “It’s more time consuming. Their breaking it down into more parts... definitely it’s harder for them and how long it takes them to solve the problem with the different methods that they’re using...they find out on their own that they’re not as efficient.” • “even knowing how, if there’s a fact that they don’t know, how to solve that fact. The different procedures...from using anything...from manipulatives...to tallies, to something that’s going to make them understand it |
|--|--|

Students in 4LC’s class “feel like peer pressure unless they know [a fact]” and the use of other strategies appears not to help students develop conceptual understanding, but is simply “something that’s going to make them understand it faster.” 4LC did not mention number sense, place value, equal groups, arrays, or area models—as described in the

standard—but focused almost exclusively on knowing basic facts as the prior knowledge needed to meet the standard. 4LC stated, “I always relate word problems to real life...everything’s practical and I want them to understand what is the purpose to solving this problem.” Regarding whether students who have not yet mastered skills can solve word problems, teacher 4LC replied, “I think they can solve it, but they definitely see the benefits of improving their facts and mastering their facts.” The stress on basic fact fluency aligns with 4LC’s overall mean score of 2.88, indicating low-constructivist beliefs, as well as nearly all responses to specific survey items related to this belief factor.

Capraro (2001) found six survey items to be highly saturated in this belief factor. 4LC indicted non-constructivist beliefs on 83.33% of these items about the relationship among skills, understanding, and word problem solving or sequencing of mathematics topics. Specific item analysis indicates 4LC believes skills (basic facts and computation procedures) precede both understanding and word problem solving, as seen in the agreement with the following MBS statements:

Item 22: Recall of number facts should precede the development of an understanding of the related operation (addition, subtraction, multiplication, or division).

Item 23: Children will not understand an operation (addition, subtraction, multiplication, or division) until they have mastered some of the relevant number facts.

Item 1: Children should not solve word problems before they master computational procedures.

Consistent with these non-constructivist beliefs are 4LC's disagreement with these MBS statements:

Item 13: *Children should have many informal experiences solving simple word problems before they are expected to memorize number facts.*

Item 4: *Time should be spent solving simple word problems before children spend much time practicing computational procedures.*

In direct opposition to Items 22 and 23, 4LC strongly agreed with this MBS statement:

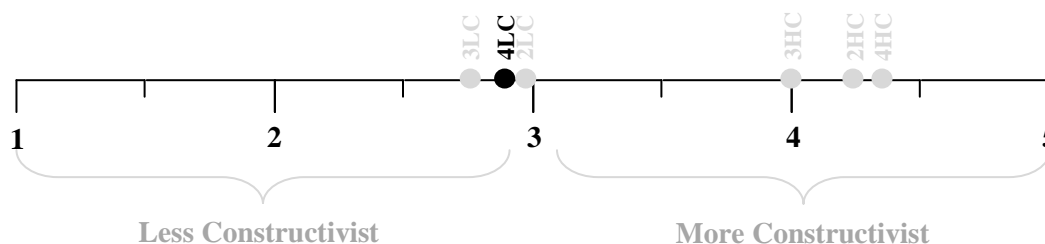
Item 6: *Children should understand the meaning of an operation (addition, subtraction, multiplication, or division) before they memorize number facts.*

Item 25: *Children should understand computational procedures before they master them.*

Despite the occasional inconsistent survey response and a few interview responses that contain elements of constructivism, 4LC's beliefs about mathematics (indicated by both overall mean score as well as specific item analysis) and implementation practices reflect a traditional approach to helping students learn multi-digit multiplication by mastering basics facts and using traditional procedures to meet this grade level standard.

Beliefs about Learning Mathematics. Figure 23 summarizes 4LC's interview data related to this belief factor. Teacher 4LC described implementation practices relating to how children best learn mathematics that are mostly non-constructivist in nature, aligning with 4LC's overall mean score of 2.88, indicating beliefs that trend slightly toward the traditional end of the spectrum. "Everything is broken down into parts...and steps... and the reasoning behind why we're doing something." While helping students

Figure 23

Teacher 4LC: CCSSM Implementation and Beliefs about Learning Mathematics**4LC Implementation Practices**

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| <ul style="list-style-type: none"> the traditional method “We go over it and over it” despite spiraling curriculum design—for retention “We go through the steps very slowly. I break it down into parts.” “Everything is broken down into parts...and steps” “It’s teacher demonstration. It’s student participation.” “It’s not all teacher directed. The students are participating. They’re coming up to the board.” “there’s accountability for each student” unit assessments, quizzes, written work games based on sequencing steps of traditional method partial products not emphasized lattice method | <ul style="list-style-type: none"> “Everything is broken down into parts...and steps... and the reasoning behind why we’re doing something.” “It’s teacher demonstration. It’s student participation.” “It’s not all teacher directed. The students are participating. They’re coming up to the board.” partial products lattice method |
|--|--|

understand the mathematical meaning behind the procedure is constructivist in nature, the pedagogical approach of constantly breaking problems down into smaller parts, which are then related to basic facts, is indicative of traditional behavioral learning theory. The emphasis on the traditional method of multi-digit multiplication was again stressed and even the active learning games described were based on figuring out the sequence of steps in the standard algorithm. Teacher 4LC’s comments regarding lattice multiplication

presented a challenge to the researcher; these comments were categorized as both traditional and constructivist. The recognition that other methods exist to find products, including lattice, reflects a more constructivist view, but the lattice method of multiplication is inherently traditional in nature—compartmentalizing the problem into basic facts using a spatial arrangement that does not facilitate a conceptual understanding of the product. Teacher 4LC suggested that lattice multiplication is not viewed favorably in the higher grade levels, yet almost requires students to use the standard algorithm that is similar in nature. The researcher used specific item analysis to examine responses related to classroom implementation of the standard within this belief factor.

Regarding how children best learn mathematics, 4LC agreed with the following MBS statement:

Item 16: *Most young children have to be shown how to solve simple word problems.*

Consistent with a traditional view of learning, 4LC disagreed with these MBS statements:

Item 28: *Children should be allowed to invent new ways to solve simple word problems before the teacher demonstrates how to solve them.*

Item 36: *Most young children can figure out a way to solve many mathematics problems without any adult help.*

However, within this belief factor, several of 4LC's responses are in direct opposition to these two responses and, instead, align with (or were at least neutral to) constructivist learning beliefs, including agreement with the following MBS statements:

Item 27: *It is important for a child to discover how to solve simple word problems for him/herself.*

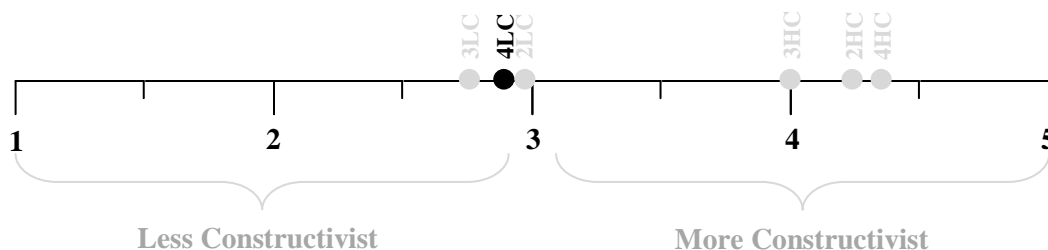
Item 33: *Children can figure out ways to solve many math problems without formal instruction.*

Item 41: *Given appropriate materials, children can create meaningful procedures for computation.*

Teacher 4LC's results indicate both constructivist and non-constructivist beliefs about learning mathematics which is consistent with an overall mean score (2.88) close to a neutral score (3). However, 4LC's described implementation practices appear mostly non-constructivist, which are consistent with this teacher's beliefs about how mathematics should be taught as presented in the following section.

Beliefs about Teaching Mathematics. Figure 24 provides a summary of 4LC's interview data related to this belief factor. Teacher 4LC described some implementation practices that were constructivist in nature, such as encouraging student participation in addition to teacher demonstration, making sure students understand the reasoning behind the procedural steps, and using student achievement to guide instructional decisions. However, each these responses also suggests a traditional teaching perspective. Student participation that involves going up to the board to follow the steps prescribed by the teacher or playing a game based on sequencing the steps of the traditional method of multiplication indicates a teacher-directed learning environment. Past student success on basic fact timed tests and competency with the standard algorithm may demonstrate procedural knowledge, but not necessarily conceptual understanding of multiplication.

Figure 24

Teacher 4LC: CCSSM Implementation and Beliefs about Teaching Mathematics**4LC Implementation Practices**

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|---|---|
| <ul style="list-style-type: none"> • confidence in traditional approach based on previous success; 95% know facts, fifth grade teachers say students are prepared • “The methods I’ve used have been successful. They’ve been very successful over the many years that I’ve been teaching it.” • “We go over it and over it. Even though, many times ... the program that we’re using is designed to teach a different skill each day. I’m one who likes the students to retain the information, so if I have to go over it for several days, um, several weeks, until they understand the material. • “We go through the steps very slowly. I break it down into parts.” • “Everything is broken down into parts...and steps” • “It’s teacher demonstration. It’s student participation.” • games using steps of traditional algorithm • rigorous practice of basic facts and computation procedures • traditional method: “that’s something they use over and over again each day. They’re almost required to use it.” • basic fact timed tests; accountability | <ul style="list-style-type: none"> • enjoyment: “It’s great. Multiplication is something I enjoy. Multiplication is something I feel I’m successful at teaching to the students. • “My role is to introduce them to the standard and not only can they apply the standard to word problems, to everyday life certain situation, but also that they retain the material for future years that they’re going to be going to school. So that, when they leave here, they understand I can solve that two-digit by two-digit problem or two-digit by three...they understand that and they can use that material. • “The methods I’ve used have been successful. They’ve been very successful over the many years that I’ve been teaching it.” • student coming to board • different means of assessment • student explanations of answer • use of manipulatives, games • reasoning behind procedural steps |
|---|---|

Teacher 4LC describes teaching students to “go over it and over” the traditional method, stating that “they’re almost required to use it.” The majority of implementation practices

related to beliefs about teaching mathematics reflect traditional beliefs about efficacy through a teaching-as-telling instructional model (Smith, 1996). These beliefs are indicated by 4LC's agreement with the following MBS statements:

Item 14: *An effective teacher demonstrates the right way to do a word problem.*

Item 7: *The teacher should demonstrate how to solve simple word problems before children are allowed to solve word problems.*

In direct opposition to the previous statements, 4LC agreed with these MBS statements:

Item 43: *Teachers should facilitate children's inventions of ways to solve simple word problems.*

Item 37: *Teachers should allow children to figure out their own ways to solve simple word problems.*

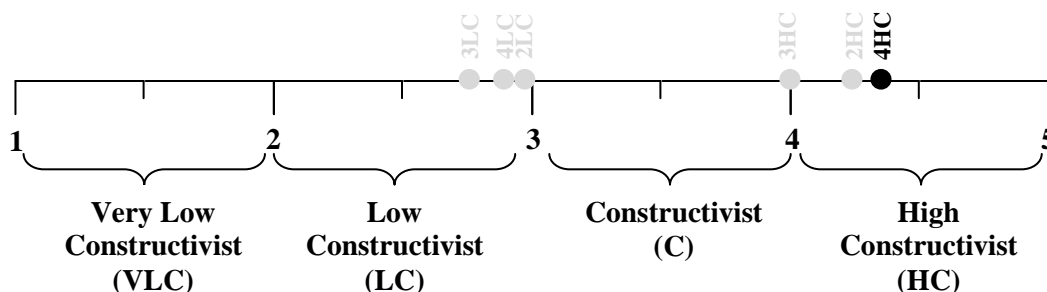
Within the belief factor, several of 4LC's responses indicate conflicting beliefs.

Implementation of the standard reflect some constructivist practices, but align more with a traditional pedagogical approach to teaching students about multi-digit multiplication: emphasis on basic facts and traditional procedures through repeated practice.

Grade 4 High Constructivist (Teacher 4HC) Profile. Teacher 4HC teaches fourth graders at elementary school 3 and has 15 years experience teaching at this grade level. Teacher 4HC earned an overall mean score of 4.35 on the MBS survey (shown in Figure 25), placing this teacher in the high-constructivist beliefs category. This was the third highest overall means score in the quantitative (n=80) data sample and the highest overall mean score of the 6 teachers interviewed.

Figure 25

Teacher 4HC: Placement on Constructivist Continuum



Teacher 4HC: CCSSM Interpretation. To open the interview, 4HC explained that “fluency would be related to math facts, if they’re fluent in their math facts.” In 4HC’s classroom, students demonstrate fluency on three-minute timed tests, but this teacher expressed concern that “the results of those don’t always reflect what the student can do.” Teacher 4HC uses oral questions to “get a better read sometimes as opposed to the assessment because I think there’s just that level of anxiety that kids feel when they’re taking something and it’s timed.” As opposed to only determining fluency by knowing a fact within a set time, 4HC stated that fluency is also “a comfort level. I think it’s just processing it and... thinking about that fact and being able to retrieve it and say it without anxiety.”

Regarding the meaning of efficiency (as used in the standard), 4HC explains “the word efficient means being able to attack a problem, like look at a problem and be able to have a way of computing that problem that is efficient for that student that they can get through and that they completely understand.” This response suggests that there are many ways to efficiently solve a problem. Teacher 4HC interprets the word *procedures* (plural)

in the standard to mean that “children are going to have different procedures as they solve problems.” These implementation practices suggest that 4HC does not equate efficiency with *a* particular method, but rather that it is important for each student to become efficient with whichever method makes sense to them. Taken together, these interpretations of the language in the standard align with 4HC’s constructivist beliefs as indicated by the overall mean score of 4.35 on the survey.

Specific item analysis is consistent both with 4HC’s comments regarding the importance of having students learn their basic facts, indicated by agreement with the following survey statement:

Item 35: Frequent drills on the basic facts are essential in order for children to learn them.

This response was one of only two items on the entire 48-item survey answered in a non-constructivist way. However, 4HC also agreed with this MBS statement:

Item 2: Teachers should encourage children to find their own solutions to math problems even if they are inefficient.

During the interview, 4HC acknowledged the limitations of using basic fact drills, such as timed tests, and described how students also learn math facts by using manipulatives and solving problems in their own way. This belief in the value of invented or alternate algorithms aligns with 4HC’s overall mean score of 4.35 and the described implementation practices presented in the following section.

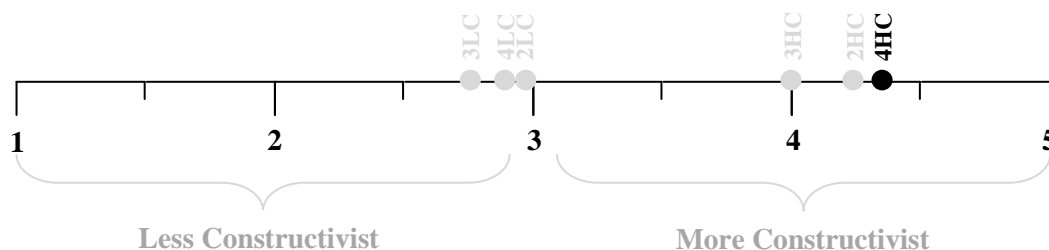
Teacher 4HC: CCSSM Implementation.

Beliefs about Mathematics. Figure 26 summarizes 4HC's interview data related to this belief factor. Teacher 4HC's responses make it clear that basic fact fluency, computational procedures, and conceptual understanding of multiplication are integrated in the classroom. For some of 4HC's students, "knowing their multiplication facts helps tremendously. Absolutely. I just see that the kids that do, tend to understand the procedures better because they're not putting so much thought into what six times seven is. They're really thinking about the procedure that they have in front of them." For this teacher, knowing facts not only makes computation procedures more efficient, but enables students to conceptually understand the procedure. While this may suggest that basic facts be mastered first, 4HC also recognizes that students who do not know their facts can still successfully solve problems, stating "If you give them the manipulatives to solve a problem, they can find a way to solve the problem." Referring to one student's struggle with math facts, 4HC explains "I have a student this year who really struggles with multiplication but it either takes him some time, or he will actually use what he knows about multiplication—and either use repeated addition or he'll draw a little picture on the side and he gets there. So, can you get there without knowing your facts? Absolutely." Specific item analysis indicates 4HC's beliefs are very constructivist within this belief factor.

Capraro (2001) found six items to be highly saturated in this belief factor. 4HC strongly disagreed or disagreed with 100% of the non-constructivist items related to the relationship among skills, understanding, and word problem solving or sequencing of

Figure 26

Teacher 4HC: CCSSM Implementation and Beliefs about Mathematics



4HC Implementation Practices

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|---|---|
| <ul style="list-style-type: none"> • basic facts: help tremendously • “problem solving becomes more fluid when the child can retrieve their facts quickly.” • “I have had students where facts do get in the way because they’re really focusing in on those facts and they’re really missing the idea behind...what’s really going on.” | <ul style="list-style-type: none"> • “Place value—absolutely. They have to know place value.” • “have to know what numbers are worth.” • “I think they have to know multiplication... they don’t always...I have kids that can go through it” • “In terms of problem solving, I feel the kids can, if you give them the manipulatives to solve a problem, they can find a way to solve the problem if they have the manipulatives in front of them. • “forget about some of these big numbers and use smaller numbers first” • multiplication as repeated addition • “think about your totals and parts” • student who struggles with facts still has success: takes some time, but uses what he knows about multiplication, uses repeated addition, draws a little picture, “able to understand the procedure because we went through it in the ways that we did. You know, we really built upon everything. So, the facts didn’t get in the way” • “So, can you get there without knowing your facts? Absolutely.” • the kids that know facts understand procedures better • kids who are not proficient sometimes try to memorize the steps and then they can’t remember, it’s not conceptual.” |
|---|---|

mathematics topics. For example, 4HC's strongly disagreed with the following MBS statements:

Item 22: *Recall of number facts should precede the development of an understanding of the related operation (addition, subtraction, multiplication, or division).*

Item 23: *Children will not understand an operation (addition, subtraction, multiplication, or division) until they have mastered some of the relevant number facts.*

Item 29: *Time should be spent practicing computational procedures before children are expected to understand the procedures.*

Item 39: *Children should not solve simple word problems until they have mastered some number facts.*

Item 47: *Children should master computational procedures before they are expected to understand how those procedures work.*

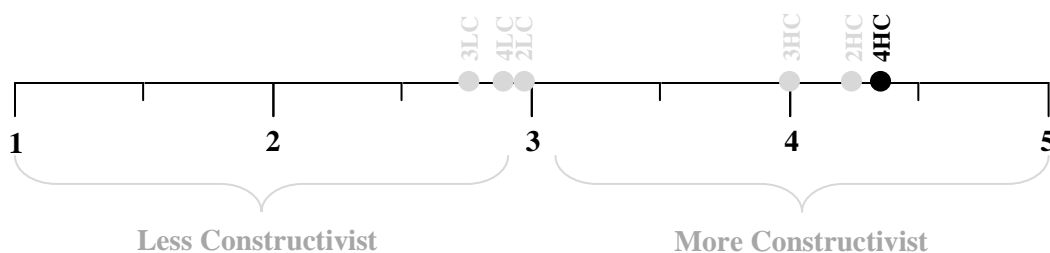
In fact, 4HC so strongly disagreed with Item 29, this teacher wrote a note under the statement: "Understand [procedures] first through manipulatives and discussion."

Similarly, 4HC agreed or strongly agreed with constructivist items in this belief factor, indicating consistency in beliefs as well as in described classroom implementation practices when helping students learn multi-digit multiplication by connecting facts and procedures with conceptual understanding and word problem solving.

Beliefs about Learning Mathematics. Figure 27 provides a summary of 4HC's interview data related to this belief factor.

Figure 27

Teacher 4HC: CCSSM Implementation and Beliefs about Learning Mathematics



4HC Implementation Practices

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|--|---|
| <ul style="list-style-type: none"> • district policy: “have to teach partial product” • lattice method in curriculum, but don’t use: inefficient and spatially difficult to set up | <ul style="list-style-type: none"> • partial product • district policy: partial product because it helps develop conceptual understanding of multiplication • concern that some teachers are teaching traditional method early • array models • “let students explore and play with that just so they understand” • “I just give them, um, DigiBlocks and so one of the first things we do is I say, okay, show me what you think multiplication really is.” • “Throughout the year I’m constantly developing that idea of parts and totals because I think that’s a huge foundation in math. So we’re kind of going with those ideas in addition and subtraction and then we...relate those to the discoveries that they have with the DigiBlocks.” • show multiplication with Digi-Blocks using smaller numbers • “Are we dealing with parts or totals...and how do we know that?” • conceptual understanding of operation • “They begin to see the relation to addition because you’re using two parts and you’re getting to a total.” • difference between addition and multiplication problems though related • then look at algorithm while using manipulatives, connecting steps of procedure to actions with manipulatives • teacher records student observations • explaining thinking and solution |
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Teacher 4HC provided a detailed description of highly constructivist implementation practices that serve as a real-life illustrations for Smith's (1996) reform model of teacher efficacy in which teachers create a learning environment that allows students to actively construct mathematical knowledge. 4HC described inquiry-based learning with questions such as "You multiplied the ones by the ones. Could you have started with the tens? Why or why not?" and "Tell how you got this partial product? What does [partial product] even mean?" are designed to elicit students' thinking about the mathematics behind the procedure.

Teacher 4HC introduces multiplication by asking students to show what they think it is using Digi-Blocks and stated:

We always start with manipulatives. We start there so they understand what they're doing... It takes us, I would say, one or two days before we even pick up pencil and paper. So they have to see those procedures in action with manipulatives. Then once we get to paper and pencil, I really do it as a group because we go step by step and then relate every procedure that we're doing to the manipulative.

These class discussions help students see larger mathematical ideas, such as "the idea of parts and totals," so students "begin to see the relation to addition because you're using two parts and you're getting to a total."

These described implementation practices for this multi-digit multiplication standard are consistent with 4HC's overall mean score of 4.35, indicating high-

constructivist beliefs. Specific item analysis yielded similar results for responses within this belief factor.

Regarding how children best learn mathematics, 4HC strongly disagreed or disagreed with all non-constructivist items but one, including these three MBS statements:

Item 15: *Children should be told to solve problems the way the teacher has taught them.*

Item 16: *Most young children have to be shown how to solve simple word problems.*

Item 26: *Children learn mathematics best from teachers' demonstrations and explanations.*

The only statement related to this belief factor with which 4HC offered a traditional response was agreement with this MBS statement:

Item 11: *It is important for a child to be good listener in order to learn how to do mathematics.*

The consistency of constructivist beliefs about how children learn mathematics combined with 4HC's implementation practices (involving class discussion in which the teacher asks questions and records students' observations and discoveries) suggest an interpretation of this survey item to include students listening to one another, not just to the teacher, to learn how to do mathematics. 4HC's stated, "We let students explore and play with that just so they understand...it's another way for them to understand the

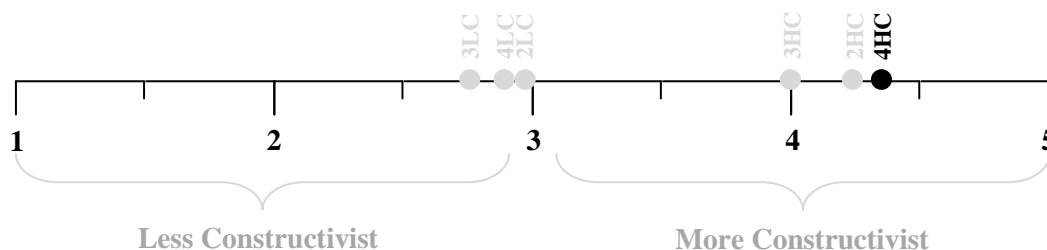
multiplication.” This response lends insight not only into this teacher’s beliefs about how students learn mathematics, but also beliefs about how it should be taught.

Beliefs about Teaching Mathematics. Figure 28 summarizes 4HC’s interview data related to this belief factor. Teacher 4HC described implementation practices that were constructivist in nature, such as asking students questions and recording their thinking, relating the operations (“multiplication is really repeated addition”), encouraging multiple voices in the mathematics classroom, and using manipulatives to promote conceptual understanding of multiplication as an operation and solve multiplication problems. The focus on one algorithm is a typically a traditional instructional model. However the focus on partial products in this case is an alternate, not standard algorithm. 4HC consistently expressed the importance of “getting them to understand that you’re using place value and getting parts of products so that we can eventually get to our full product” and even wanted students to think about what the name partial products means. Understanding the distributive property in multiplication can facilitate the later understanding of the distributive property in algebra. The importance of helping students develop a strong mathematics foundation was reflected throughout the interview, including comments about making connections between “foundational” mathematical ideas (parts and totals) and bringing in “the historical perspective on [mathematics] which I think is really, really important. And what number is and how it even evolved over time and how it ... came about.”

Teacher 4HC’s described implementation practices related to beliefs about teaching mathematics reflect constructivist beliefs that align with the overall mean score

Figure 28

Teacher 4HC: CCSSM Implementation and Beliefs about Teaching Mathematics



4HC Implementation Practices

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|---|--|
| <ul style="list-style-type: none"> • follow district mandate of teaching one algorithm | <ul style="list-style-type: none"> • inquiry-based instruction • teacher-as-facilitator • guided discovery • teacher for conceptual understanding • “let students explore and play with that just so they understand” • help students understand place value: “huge emphasis on it” • historical perspective: “what number is and how it even evolved over time” • “My role is to really place a lot of emphasis on place value...for them to understand what that really means. So that when you do those procedures, all those connections are made.” • help make connections in math: multiplication is really repeated addition • how to use what you know and break apart numbers to solve problems • “I’m just recording...I want to take out that idea of procedure and just going through steps. I just still want them to do the thinking and have that freedom to do their own thinking, to kind of talk and work together as a group” • “We go from really just using the manipulatives and that’s it. And then we go into me recording and saying, ‘I just want to record the thinking that I’m seeing...and this is how I’m going to record.’” • using the same mathematical language through process of using manipulatives |
|---|--|

of 4.35 as well as responses within this belief factor, including strong agreement with the two following MBS statements:

Item 9: *Mathematics should be presented to children in such a way that they can discover relationships for themselves.*

Item 43: *Teachers should facilitate children's inventions of ways to solve simple word problems.*

All of 4HC's survey responses reflected constructivist beliefs about how mathematics should be taught. When implementing the fourth grade multi-digit multiplication standard, 4HC describes the teaching role as a facilitator: "I want to take out that idea of procedure and just going through steps. I just still want them to do the thinking and have that freedom to do their own thinking, to kind of talk and work together as a group."

Comparative Analysis of 4LC and 4HC. The analysis between these two fourth grade teachers finds that mathematics beliefs influence teacher practices related to the CCSSM, from shaping how teachers interpret the language of the standard to the actions they take when working with students in the classroom. One critical example from the data illustrates how differences in mathematics beliefs influence two teachers to implement the same CCSSM in very different ways.

Both teachers 4LC and 4HC acknowledge the importance of basic fact fluency in learning to multiply multi-digit numbers and each focuses on a single algorithm. However, the first teacher identified only basic facts as the prior knowledge needed to meet the standard, stating, "They need to know their math facts. They cannot be

successful unless they know their math facts.” Teacher 4LC uses daily practice and timed tests because “It’s one of the most important things that a student should know are their facts.” Teacher 4HC included basic facts as one of several types of prior knowledge needed to multiply multi-digit numbers, but emphasized place value, number sense, and a conceptual understanding of multiplication as an operation first.

These differences in beliefs about mathematics (the relationship among skills, understanding, and word problem solving as well as the sequencing of mathematics topics) influence them to adopt very different implementation practices in the classroom. Teacher 4LC places such a strong emphasis on the traditional method of multiplication that students learn through rigorous practice as they use it “over and over again each day. They’re almost required to use it.” Teacher 4HC focuses on the partial product method of multiplication, always starting with manipulatives and spends days “let[ting] students explore and play with that just so they understand” the concept of multiplication. Teacher 4HC explains, “I want to take out that idea of procedure and just going through steps. I just still want them to do the thinking and have that freedom to do their own thinking, to kind of talk and work together as a group” so that students “see those procedures in action with manipulatives.”

The following MBS statement generated very different responses from 4LC and 4HC:

Item 29: Time should be spent practicing computational procedures before children are expected to understand the procedures.

Teacher 4LC agreed with this statement while 4HC strongly disagreed, even writing in a note stating “understand first through manipulatives and discussion.” These differences in beliefs influence differences in implementation practices related to the same Grade 4 CCSSM mathematics standard.

Phase II Summary

Comparative Analysis of LC Teachers and HC Teachers. A synthesis of quantitative and qualitative results from individual teacher data and within grade level comparison data generated the following key findings to answer the second research question regarding how mathematics beliefs influence the way teachers interpret and implement mathematics standards in the classroom.

The three teachers categorized as holding low-constructivist beliefs earned overall mean scores of 2.94 (2LC), 2.73 (3LC), and 2.88 (4LC). Each of these scores lies very close to the neutral score of 3 with distances equal to 0.06 (2LC), 0.27 (3LC), and 0.12 (4LC). Specific item analysis indicated these near neutral overall mean scores were the result of three factors.

First, the LC teachers offered more neutral responses than their HC teacher counterparts. Teachers 3LC and 4LC each responded neutrally to 10 MBS items (20.83%) while 2LC responded neutrally to 11 MBS items (22.92%). More neutral responses yield a greater number of response values equal to 3.

Second, the LC teachers were not as willing to offer a strong response in either direction. Teacher 2LC *strongly agreed* or *strongly disagreed* with 4 MBS items (8.33%). Teacher 3LC offered a *strongly agree* or *strongly disagree* to 5 MBS items (10.42%) and

4LC offered only 2 strong responses on the entire 48-item survey (4.17%). Strong responses translate to numerical scores of 5 (constructivist) or 1 (traditional). Fewer strong responses result in fewer extreme scoring values, rendering an overall mean score closer to 3.

Third, the LC teachers offered contradictory responses, sometimes giving a constructivist response and other times giving a traditional response—even on closely related (by meaning) statements. When contradictory responses are given, the numeric values cancel one another. Each of these three factors resulted in the LC teachers earning overall mean scores near the neutral score of 3, but categorized as low-constructivist by trending toward the traditional end of the continuum.

By contrast, the three teachers categorized as high-constructivist earned overall mean scores of 4.21 (2HC), 4.02 (3HC), and 4.35 (4HC). Each of these scores lies farther from the neutral score of 3 than the scores of the LC teachers, with distances equal to 1.21 (2HC), 1.02 (3HC), and 1.35 (4HC). Specific item analysis indicated these mean scores reflected different results related to the same three factors.

First, the HC teachers offered fewer neutral responses: 4 MBS items (8.33%) answered neutrally by each HC teacher. Fewer neutral responses mean fewer numerical values equal to the neutral score of 3.

Second, the HC teachers offered more *strongly agree* or *strongly disagree* responses to survey items. Teacher 2HC gave strong responses to 18 MBS items (37.5%). Teacher 3HC *strongly agreed* or *strongly disagreed* with 13 MBS items (27.08%).

Teacher 4HC offered strong responses on 25 MBS items (52.08%) which was the highest number in the sample.

Third, the HC teachers offered fewer contradictory responses to closely related (by meaning) statements. The combination of these last two factors made a significant impact on overall mean scores. Consistently constructivist responses translate to numerical values of 4 and 5, moving the overall mean score toward 5 and away from the neutral score of 3. Together, these three factors resulted in the HC teachers earning overall mean scores, categorizing them as high-constructivist.

The differences in the strength of mathematics beliefs on the survey aligned with the interview data. LC teachers described interpretation and implementation practices that were both constructivist and non-constructivist. HC teachers, who offered stronger beliefs on the survey, consistently described constructivist interpretation and implementation practices. This strength (or lack thereof) in beliefs was reflected in two distinct examples. Teacher 3LC stated “this was something in the survey that it was like a toss-up for me” whereas 4HC added hand-written notes to support *strongly agree* responses. Clearly, stronger and more consistent beliefs influence more consistent practices while neutral or contradictory beliefs influence varied practices.

Conclusion

The mixed methods research design enabled quantitative data that answered the first question (*What do teachers believe about mathematics, teaching mathematics, and learning mathematics?*) and qualitative data that answered the second question (*How do these teacher beliefs influence their interpretation and implementation of the Common*

Core State Standards for Mathematics?). The survey results indicated that teachers—even those who use the same curriculum, and work in the same district in similar schools with similar student populations—have different mathematical beliefs. The interview results indicate that beliefs influence practice, confirm that differences in teacher beliefs yield differences in teacher practices, and allowed teachers to explain how beliefs influence practices in their own words. The same mathematics standard looks different in the hands of each teacher. Thus, a common standard does not always result in a common learning experience for students. The final chapter will discuss the professional implications of these findings, acknowledge the limitations of the study, and present opportunities for further research in the mathematics education field.

CHAPTER FIVE: CONCLUSIONS AND RECOMMENDATIONS

“From the onset of a study, the question one chooses to ask and the data that one chooses to gather have a fundamental impact on the conclusions that can be drawn.”

(Schoenfeld, 2007, p. 70)

Introduction

To investigate the beliefs-practice relationship related to the Common Core State Standards for Mathematics (CCSSM) (NGA, CCSSO, 2010), this study sought answers to two research questions. The first question asked what teachers believe about mathematics, teaching mathematics, and learning mathematics. The second question asked how those beliefs influenced the ways teachers interpret and implement the CCSSM. The previous chapter presented the quantitative results of the Mathematics Beliefs Scales (MBS) (Fennema et al., 1990) survey used to gather data from a sample of 80 K-6 teachers to answer the first question as well as the qualitative results of personal interviews with a smaller number of teachers (n=6) drawn from the original sample.

While it is widely accepted that teacher beliefs influence their professional practice, there remain significant gaps in knowledge within this area of study. One challenge to researchers is that beliefs are an inherently “messy construct” (Pajares, 1992). Early efforts in the field of mathematics education research involved examining teacher beliefs about mathematics in general or about a mathematics curriculum in particular, but did not investigate teacher beliefs about the teaching and learning of specific mathematics content (Peterson et al., 1989). The mathematics education reforms of the 1990s provided an opportunity for researchers to address this gap in knowledge;

many studies began to investigate the beliefs-practice relationship related to mathematics standards, reform efforts, curricula, and children's mathematical thinking.

The purpose of this study was to build on this existing body of research by applying previously used methodologies and instruments to examine beliefs and practices in a new era of educational reform created by the widespread adoption of the CCSSM. The CCSSM are “learning goals [that] outline what a student should know and be able to do at the end of each grade” and were created and adopted “to ensure that all students graduate from high school with the skills and knowledge necessary to succeed in college, career, and life, regardless of where they live” (NGA & CCSSO, 2010, “Development Process”). Historically, the CCSSM offer mathematics education researchers a unique opportunity; for the first time in the history of the United States, students in nearly all states and territories have a consistent set of mathematics standards. From the literature, we know that beliefs are both filters through which new information—like mathematics standards—is interpreted (Pajares, 1992) and dispositions to action (Philipp, 2007). This study was designed to explore whether teachers who hold different beliefs (about mathematics, how students learn mathematics, and how to best teach mathematics) interpret and implement the CCSSM in different ways.

This chapter offers conclusions that are fundamentally impacted by both the research questions asked and the specific data gathered and analyzed in this study. These conclusions reflect the findings presented in chapter 4 in relation to the specific purposes of this study, existing knowledge in the literature, and the wider mathematics education

field. This final chapter will also address the limitations of the research and offer recommendations for future study on the topic.

Summary of Results

Using the MBS instrument, the survey conducted during the first research phase, the researcher translated teachers' responses into numerical values to reveal differences in beliefs among a relatively homogenous sample (n=80) of teachers. Beliefs were organized using a three-factor framework: beliefs about mathematics (the relationship among skills, understanding, and word problem solving as well as the sequencing of mathematical topics), beliefs about how children learn mathematics, and beliefs about how mathematics should be taught. The researcher used survey data to place and categorize participants on a constructivist continuum from very low constructivist to high constructivist, select interviewees, and refine interview questions.

The survey results showed teachers in each of the eight grade levels (K—6 and Multiple) held different mathematics beliefs. Grade levels 2, 3, and 4 reflected the greatest differences in beliefs between the lowest scoring and highest scoring teachers; these 6 teachers were selected to participate in the second research phase. The data also revealed specific survey items with the greatest variation in response. The content and language of these survey statements informed the final version of interview questions, guided follow-up questions, and presented opportunities for phase one and phase two data integration.

The qualitative research phase asked teachers to explain their interpretation of specific language used in a particular grade level CCSSM standard and describe how they

implement the standard with students in the classroom. These implementation practices related not only to the context of the specific standard, but also enabled the researcher to better understand six teachers' beliefs about mathematics, how students learn mathematics, and how mathematics should be taught. The researcher classified the practices described by each teacher as less-constructivist or more-constructivist. The researcher integrated interview data and survey data to gain a more complete understanding of the beliefs-practice relationship for each individual teacher, compare same-grade teachers, and compare Low Constructivist (LC) and High Constructivist (HC) teachers. The interview and integrated data results showed that teachers interpret and implement mathematics standards that align with their personal mathematics beliefs. Thus, differences in beliefs translate to different instructional practices.

Discussion of Results

Strength of Beliefs

Three of the six teachers interviewed placed in the Low Constructivist (LC) category based on overall mean survey scores of 2.73 (Second Grade – 2LC), 2.94 (Third Grade – 3LC), and 2.88 (Fourth Grade – 4LC). The LC teachers all earned scores close to the neutral score of 3. The three teachers categorized as High Constructivist (HC), based on overall mean survey scores of 4.21 (Second Grade – 2HC), 4.02 (Third Grade – 3HC), and 4.35 (Fourth Grade – 4HC). The HC teachers all earned scores farther from the neutral score of 3. Specific item analysis indicated the near-neutral overall mean scores of the LC teachers were the result of two factors related to the strength of beliefs: more neutral responses and fewer strong responses.

First, the LC teachers offered a neutral response to 10 items=20.83% (Grades 3 and 4) or 11 items=22.92% (Grade 2) MBS items. On the survey, these teachers were not willing to agree or disagree with approximately one-fifth of survey statements. The HC teachers, however, felt strongly enough to either agree or disagree with the survey statements, each offering a neutral response to only 4 survey items=8.33%. Second, the LC teachers offered few *strongly agree* or *strongly disagree* responses to MBS items: 4 items=8.33% (Grade 2), 5=10.42% (Grade 3), and 2=4.17% (Grade 4). The LC teachers were not willing to commit to a strong response, indicating more centrist beliefs. By contrast, the three HC teachers offered more *strongly agree* or *strongly disagree* responses to survey items: 18 items=37.5% (2HC), 13 items=27.08% (3HC) and 25=52.08% items (4HC).

There are many reasons why LC teachers earned low-constructivist, yet near-neutral scores. One possible explanation for this result is that these LC teachers hold balanced views that incorporate both traditional and constructivist beliefs; they were not willing to express a belief that was clearly traditional or constructivist in nature. A second possibility is that the LC teachers were not sure how to respond. In a Likert-style survey, choosing a neutral response could reflect the decision to not make a choice at all, as supported by 3LC's comment that some survey items were a "toss up." A third possibility that must also be considered is the current climate in the mathematics education field. Post (1988) found "the vast majority (96 percent) of professional mathematics educators describe their primary philosophical orientation as belonging squarely in the cognitive camp" (p. 27). Perhaps teachers feel uncomfortable expressing traditional mathematics

beliefs, aligned with the behavior camp, and prefer to appear neutral rather than openly communicate an unpopular view. Interview comments suggest the LC teachers use the learning of skills, such as basic facts and computation procedures, and the formal structure of mathematics content to drive instruction.

These explanations are not applicable to the high-constructivist, farther from neutral, scoring HC teachers. These teachers appear more committed to their constructivist beliefs system, as suggested by the strong agreement with constructivist statements and strong disagreement with traditional statements. Teacher 4HC went so far as to write in comments to support strong responses. The HC teachers provided reasoning to support their instructional approaches and openly used language associated with constructivist learning theory: discovery, inquiry, exploration, invention, discussion, and using students' thinking to drive instruction. In the current climate, these educational strategies are widely accepted as best practices and would likely generate broad support from teaching colleagues.

Conflicting Beliefs

The near-neutral overall mean survey scores earned by the LC teachers is also the result of a third factor: contradictory responses. On a Likert-type survey, near-neutral overall mean scores can also be obtained by offering responses that directly (two items with slightly different wording) or indirectly (choosing some items with low values and others with high values across the entire survey) cancel numerically. The three LC teachers offered survey responses that were directly or indirectly contradictory within and

across all three belief factors. The three HC teachers, however, offered consistently constructivist responses within and across belief factors.

Interview transcripts supported this lack of consistency in beliefs of the LC teachers. Teacher 2LC believed real-world word problems based on life experiences could help students learn facts and procedures, but since not all students had such experiences, the facts and procedures should come first. Teacher 3LC believes students can understand and solve word problems before facts are mastered, yet would “never teach word problems first.” In a striking example of direct contradiction, 4LC agreed or strongly agreed with all four of these survey statements:

Item 22: Recall of number facts should precede the development of an understanding of the related operation (addition, subtraction, multiplication, or division).

Item 23: Children will not understand an operation (addition, subtraction, multiplication, or division) until they have mastered some of the relevant number facts.

Item 6: Children should understand the meaning of an operation (addition, subtraction, multiplication, or division) before they memorize number facts.

Item 25: Children should understand computational procedures before they master them.

Clearly it is not possible that basic fact mastery precede the understanding of operation (Items 22 and 23) *and* understanding of operation and computational procedures precede basic fact mastery; it is not possible that *both* A must precede B *and* B must precede A.

Such examples of opposing beliefs were offered by all three LC teachers on the surveys and during the interviews. Some possible explanations for the lack of consistency is that the survey items were not read carefully, survey items were misinterpreted, or the teacher changed his or her mind when survey item wording was slightly altered.

By contrast, the three HC teachers offered little to no contradictory responses. In the rare case where conflicting responses were expressed, interview comments helped explain any differences. For example, 4HC agreed with this survey statement:

Item 11: *It is important for a child to be good listener in order to learn how to do mathematics.*

This particular response could suggest a traditional belief that students play a passive role, learning by listening to the teacher's explanations and demonstrations. However, this teacher described several practices that refute this suggestion; devoting considerable time to whole class and small group discussion and encouraging students to learn by listening to each other as well as the teacher.

Fidelity of CCSSM Interpretation and Implementation

The LC teachers described practices that were mostly traditional in nature, but also included some constructivist elements. The LC teachers are incorporating constructivist classroom practices required by the standard, but to a lesser degree due to their traditional beliefs. This may be similar to the fidelity of implementation discussed by Brown et al. (2009), but related to implementation of standards instead of curriculum. If the LC teachers hold beliefs that are in conflict with the underlying beliefs of the CCSSM, there will be less faithfulness between the standards to be implemented and the

actions taken by the implementer. The HC teachers, whose constructivist beliefs are consistent with beliefs underlying the CCSSM, will demonstrate a more faithful implementation as seen in the detailed descriptions of highly constructivist classroom practices.

This difference in degree of fidelity of implementation of the CCSSM, seen in many teacher responses, is highlighted in Grade 2. The language in the Grade 2 standard specifically includes the use of models and an understanding of place value to add and subtract numbers within 1000. Teacher 2LC described the use of manipulatives and one model, the T-chart, to help students add and subtract by considering place value. However, this teacher first models exactly how to set up the T-chart, requires students to begin in the ones place, and then hangs the chart in the room so that students can refer to it and remember *the* way to add and subtract. While models, drawings, and strategies are present in the classroom, the students do not appear to generate their own models and strategies; they copy the model demonstrated by the teacher. These described practices do not reflect faithful implementation of the written and intended standard.

Teacher 2HC, by contrast, interprets and implements this same standard differently. This teacher described how students begin by using manipulatives to explore how to calculate answers in a variety of ways. Using guiding questions, the teacher asks students to discuss how they solved the problem, explaining what worked, what didn't work and why or why not. Students are then encouraged to try several methods shared by their peers to solve the same problem and figure out which way works best for them. The variety of models and strategies written in the language of the standard are faithfully

enacted in this classroom. While both teachers discussed the use of the standard algorithm, the emphasis and timing are vastly different in these two classrooms.

Differences in the ways teachers interpret the language of the CCSSM standards illustrates this degree of fidelity further, particularly regarding the use of the word *efficiency* in the CCSSM language. The LC teachers appeared to equate the word *efficiency* with the traditional method of calculation. Teacher 3LC commented on the preferred use of arithmetic (as opposed to drawing or models) to solve problems and teachers 2LC and 4LC repeatedly stressed the importance of proficiency with the standard algorithm by following specific steps and procedures. This emphasis on the standard algorithm, taught through step-by-step rigorous practice and measured by a student's ability to perform the proper procedure, reflects a behavioral view of learning.

Behavioral psychology, founded on the work of Thorndike (1898), Skinner (1938), and—more recently—Gagné (1985), is based on stimulus-response theories involving operant conditioning where “appropriate behavior is gradually ‘shaped’ into the desired outcome” (Post, 1988, p. 2). Behavioral psychology was strongly supported by research with animals in the laboratory. In the 1960s, neo-behaviorism emerged to determine whether human and animal learning are similar. Gagne’s (1985) work focused on human learning and behavioral responses to instruction which involved arranging conditions to assure effective learning. According to Gagne’s view, if a learner is able to perform a given task (behavior), then learning has occurred. While this perspective is the basis for traditional school mathematics curricula (Post, 1988), it is not the only perspective.

The three HC teachers encouraged the use of multiple, even invented, algorithms to allow children to solve problems in ways that were not only developmentally appropriate but also enabled students to construct and discuss the mathematics concepts behind the procedures. This emphasis on knowledge construction and socio-cultural aspects of human learning reflects a cognitive perspective influenced by the work of Piaget (1950/1970), Vygotsky (1930/1978), and others postulating the constructivist learning theory.

Changes in Beliefs

Another explanation for the use of both traditional and constructivist practices in the LC teachers' classroom requires a shift in perspective. This study focused on the ways beliefs influence practice, but researchers have also found evidence to support the view that mathematics beliefs can be changed (Ambrose, 2004; Fennema et al., 1990; Fennema, et al., 1996; Peterson et al., 1989). From this perspective, the requirement to implement more constructivist standards and the opportunity to see students learn in new ways may be slowly changing these teachers' mathematics beliefs. Despite directly observing students who have not mastered basic facts but nevertheless effectively solve problems by using models and drawings, teacher 3LC continues to make instructional decisions based on the belief that facts and procedures must be mastered first. This finding confirms the pervasive and powerful nature of beliefs—even in the face of contrary evidence (Munby, 1982). Yet, this is 3LC's first year teaching. Only time will tell if more experiences with children in the classroom will lead to a change in beliefs. While changing beliefs were not the focus in this study, it cannot be ignored that a

sustained reform effort combined with professional development opportunities (such as those available to teachers in this district) may result in teachers changing from traditional to more constructivist beliefs. However, without sustained reform efforts supported by quality professional development, research has shown that teachers may revert to previous practices (Franke et al., 2001).

Overall, the HC teachers' high-constructivist beliefs and highly constructivist interpretations and implementations, which appear to closely align with the underlying beliefs of the CCSSM, were found to be consistent within and across all three belief-factors. The LC teachers, however, were found to hold both conflicting beliefs and offered a less-than-faithful implementation of the CCSSM, perhaps reflecting professional tension with both the beliefs underlying the CCSSM and the constructivist climate of the current mathematics education field.

Limitations and Recommendations

This research was conducted to explore the beliefs-practice relationship related to the CCSSM. Although the data collected was sufficient to answer the research questions posed, there are limitations to this study that affect the generalizability of the findings in other settings across the field. The researcher acknowledges these limitations in the following sections and offers reasonable improvements to inform future studies. There are three primary limitations of this study—design, quantitative, and qualitative—that reveal such opportunities for improvement.

Design Limitations

In response to Cobb's (2007) call for researchers to act as bricoleurs who use multiple lenses through which to view constructs and employ multiple tools to answer research questions, the researcher chose an explanatory mixed methods design (Creswell, 2009) due to the complex nature of both beliefs and teaching. One limitation of this research design was the decision to not conduct classroom observations. Stepping into the classroom to watch how a particular CCSSM standard is implemented would have yielded another set of data to be used in conjunction with the survey and interview data. The proverb *actions speak louder than words* expresses the importance of seeing teachers in action to understand both beliefs and practices. Classroom observations provide a specific context in which to see teachers enacting their mathematics beliefs. Perhaps more importantly, observational data affords the opportunity for the researcher to directly experience the learning environment; how a teacher describes classroom practices may be very different than what actually occurs in the classroom. Sometimes actions are inconsistent with professed beliefs. In this study, the opportunity to see the LC teachers in particular (who offered less descriptive classroom accounts) would have added an additional means to better understand the conflicting mathematics beliefs they appear to hold. While classroom observations were not a part of this study due to resources and time constraints, future studies would be improved by triangulating three types of data: survey, interview, and observational.

Phase I: Quantitative Limitations

To answer the first research question, the researcher used the MBS survey instrument to collect quantitative data to determine what teachers believe about mathematics, teaching mathematics, and learning mathematics and reveal any differences in these beliefs.

Sample Size. The sample size for the survey was 80 teachers from one school district. Due to the sample size, the predictive power of this study across the greater teaching population is not strong. Also, because the researcher's teaching experience is at the elementary and middle school level, the teachers involved in the study were limited to Grades K—6. While the intention of the design was to select teachers from one school district to create a homogenous sample that controlled for variables (such as curriculum, district training and professional development, and year of CCSSM implementation), the predictive power of the findings could be strengthened by studies with larger sample sizes. Teachers could be selected from larger districts or across several districts. Also, since the CCSSM is for students in Grades K—12, studies could also examine beliefs and practices of teachers at the junior high and high school levels.

Instrument. The researcher chose to use an existing, valid, and reliable instrument to measure teacher beliefs. However, other researchers have shown potential weakness of using Likert-style surveys (Ambrose, et al., 2004) to capture something as complex as teacher beliefs. Surveys lack context which makes it difficult for some participants to respond accurately; they may enter neutral response or misunderstand the statement altogether. Another factor is the age of the survey. While the survey is still

appropriate for examining teacher beliefs, the language of the survey is now fifteen years old. Zollman and Mason (1992) created a new survey to measure teacher beliefs about the 1989 NCTM *Standards*. Perhaps an updated version of this survey—still designed to measure beliefs about mathematics, teaching mathematics, and learning mathematics—using language taken directly from the CCSSM would yield even stronger data.

These first two limitations, lack of context and age of the survey, are perhaps reflected in participant responses and scoring criteria for MBS Item 11: *It is important for a child to be good listener in order to learn how to do mathematics*. As previously discussed, some teachers clearly interpreted this statement to include listening to other students via classroom discourse. However, the MBS authors scored a *strongly agree* response as traditional, valued at 1 point. When the MBS was created, the typical (and traditional) mathematics classroom involved students listening to the teacher. After more than two decades of mathematics education reform, many classrooms now encourage students to learn by communicating, reasoning, justifying, and challenging ideas of fellow classmates; in other words, listening to others. There are many highly constructivist teachers who may strongly agree with this statement. Thus, the evolution of the mathematics classroom requires a change in the language or scoring of this item.

Also, previous researchers (Capraro, 2001) have identified survey fatigue as a weakness in the 48-item survey and offered a revised version consisting of 18 items. Any updated version of the survey might not only update the language of the items, but reduce the number of items as well. Finally, due to the number of neutral responses offered by the LC teachers in this study, altering the Likert-scale format of the survey to eliminate

the option of a neutral response will force teachers to decide whether they agree or disagree with a statement first and then chose the degree of their response. While the original MBS survey enabled the researcher to collect valuable data to answer the first research question, select interviewees, and refine interview questions, there is room for improvement regarding the instrument used to assess teachers' mathematics beliefs.

Phase II: Qualitative Limitations

To better understand the quantitative data and answer the second research question about the ways in which teachers interpret and implement the CCSSM, the researcher conducted personal, semi-structured interviews with six teachers from the sample. During these interviews, the researcher encouraged three pairs of same-grade teachers (Grades 2, 3, and 4) with different mathematics beliefs (as measured by the survey) to explain the meaning of the language used in the standard and describe the learning activities and environments they create to help children learn the content of a particular mathematics standard.

Sample Size. The researcher's decision to interview six classroom teachers was based primarily on feasibility. Due to the small-scope of the study, it was reasonable to conduct personal interviews with only a small number of teachers from the quantitative sample ($n=80$). The teachers interviewed were not randomly selected; they were chosen specifically for their extreme scores within their grade levels. Choosing three same-grade pairs of teachers with different mathematics beliefs allowed for within- and across-grade comparisons, but the small qualitative sample size ($n=6$) limits the predictive power of the findings. Interviewing more teacher participants across more grade levels would

allow for better predictions about the beliefs-practice relationship for teachers across the wider mathematics education field. Increasing not only the sample size, but also including teachers from the upper grade levels (Grades 7-12) would provide valuable information on the topic.

Accuracy of Interview Data. The researcher operated under the assumption that participants would offer accurate descriptions of their classroom practices. All of the participants were willing to share their views and offer glimpses into the real world of the classroom. Yet, without numerous classroom observations to ensure the accuracy of these descriptions, the researcher is left to trust these accounts. Collecting classroom observation data would afford the opportunity to triangulate the data and offer a clearer picture of the CCSSM learning environment.

Qualitative data and analysis is inherently subject to researcher bias. The phenomenon of beliefs as a filter for information is not only the subject of this study, but also shapes the research methodology. Another researcher may have asked different follow-up questions during the course of the interview, leading to a different set of collected qualitative data. Additionally, another researcher may have interpreted, coded, and categorized the textual data differently, affecting the emerging themes, results, and conclusions drawn in the study. However, the convergence of survey (numerical) data and interview (textual) data, analyzed within the framework of themes established by decades of research on mathematical beliefs using the same or similar instruments and methodologies, lends support to the accuracy of the results and conclusions made by this researcher. The opportunity to have other researchers conduct comparative analyses of

the qualitative data was not feasible for this study, but should be considered for other studies.

Future Study

This small-scale study was designed to explore the beliefs-practice relationship at a unique time in American mathematics education history. Currently, teachers in 42 states, the Department of Defense education system, Washington D.C., Guam, the Northern Mariana Islands, and the U.S. Virgin Islands teach in classrooms under the CCSSM. Of all these teachers, this researcher worked with 80 educators in one school district. The data presented here confirm previous findings about the ways teacher beliefs influence professional practice when applied to a new generation of mathematics education reform and standards. Teachers' mathematics beliefs influence how teachers interpret standards, the ways they order or integrate mathematics content, and the roles played by both teacher and learners in the mathematics classroom. However, these results and conclusions are simply a beginning; further study is needed.

Given the widespread adoption of the CCSSM and the prevalence of research during similar periods of mathematics education reform (following the New Math and NCTM *Standards* movements), there is no doubt researchers will investigate a variety of topics related to the CCSSM. Studies evaluating curriculum alignment, student achievement, and implementation costs have already begun. Yet, considering the powerful effects of mathematics beliefs, it is important not to forget McLeod's (1992) call to "integrate affective issues into studies of cognition and instruction" related to the CCSSM (p. 575).

Considering the powerful effect of beliefs on the success or failure of educational reform efforts (Ross et al., 2002; Ross et al., 2003), the researcher expects to see a wide variety of studies on this topic in coming years. The ultimate goal of educational reform, including the push toward national content standards, is improved student learning. Previous research has shown that teacher beliefs impact student learning (Carpenter, Fennema, Peterson, & Carey, 1988). Therefore, longitudinal studies that examine the relationship between student achievement and their placement in classrooms taught by teachers with various mathematics beliefs may lead to a better understanding of how teachers' beliefs influence student performance. Will students who are taught by HC teachers for several years in a row learn more? How does changing between classrooms taught by LC teachers and HC teachers affect student learning? Will students who are taught by LC teachers for several years in a row learn less? These are some of the many questions that involve the ultimate goals of educational reform efforts, yet remain unanswered.

While the teachers in this study did exhibit differences and mathematical beliefs and practices, it is important to note that even the lowest scoring teachers on the MBS were still close to the neutral score of 3, categorizing them as holding low constructivist beliefs. Given the discrepancies in practice found between the LC and HC teachers in this study, it would be interesting to investigate how teachers with very low constructivist beliefs interpret and implement the CCSSM.

In this study, the survey item analysis was limited to the 6 interview participants. In the future, the researcher may elect to review the response data for all 80 first phase

participants to determine whether findings (related to the number of neutral responses, strength of responses, and contradictory responses) for the 6 participants are consistent across the wider sample.

The study of beliefs is complex and difficult to separate from other factors that influence teaching. Conducting large studies may introduce too much variability of factors (such as differences in curriculum, administrative mandates and support, local politics, professional training and development opportunities, the influence of prior mathematics standards, and student population) to yield meaningful results. Thus, this researcher recommends several, similar, small-scale studies that investigate the differences in beliefs and practices between teachers in the same grade within the same school district. Are differences in mathematics beliefs and professional practice related to student populations or the educational beliefs held by team, school, or district leaders? A subsequent meta-analysis of results may indicate wider patterns across the greater teaching population, allowing for meaningful and generalizable conclusions.

Implications for Practice

“The widespread adoption of the Common Core State Standards for Mathematics (CCSSM) presents an unprecedented opportunity for systemic improvement in mathematics education in the United States” (NCTM, 2013, “Supporting the Common Core...”). Although focused on one facet of the CCSSM movement, the findings in this study have direct implications for professional practice. These implications for practice, in turn, promote both systemic and individual improvement in mathematics education. When districts, schools, grade levels, and teachers appreciate the impact of educational

beliefs on teachers' decisions and actions in the classroom, valuable conversations begin—conversations that include affective aspects of teaching and learning mathematics.

At the district level, teachers and administrators have spent the last several years focused on understanding the standards, training teachers about the standards, identifying changes in the grade level placement of content, adjusting curriculum to better align with the standards, administering new assessments, and determining how those assessments would be used. Now that the initial “survival” period is over, district leaders should begin to focus on affective issues related to the CCSSM. It is important that district administrators understand that even with standardization, there still exists wide variability in how mathematics content is taught throughout the district. Through district professional development opportunities, mathematics supervisors can administer beliefs assessments to help teachers become aware of their beliefs, discuss the influences of those beliefs on the learning environment, and introduce relevant research to guide the conversation.

As part of a continuous improvement model, many schools now engage in Professional Learning Communities (PLC) (DuFour, 1998). A PLC is described by DuFour (1998) as persons with expertise in a particular field who stay current by committing to ongoing study and continual practice in a group with shared interests. As part of a formal or informal community of practice, school administrators can use current research on teacher beliefs to generate professional dialogue and inform classroom practice. Introducing affective components of teaching and learning to the professional conversation may help teaching colleagues discuss differences in beliefs, the many ways

content is taught, and—perhaps more importantly—assess the strengths and limitations various pedagogical practices have on student learning. District and school leaders must encourage teachers to view themselves as learners who seek new knowledge in their professional field, including knowledge about affective influences on teaching and learning mathematics. Part of that professional conversation should include whether and how teachers’ beliefs and practices have changed during these early years of the CCSSM era.

Regardless of the district or school conversation, individual teachers demonstrate professionalism through reflection. As teachers are pressed to reveal what they believe, they can compare their beliefs against current research findings and reflect on the impact of their beliefs on teaching and learning. Research has shown that awareness of beliefs and actions enable teachers to generate their own learning (Franke, et al., 2001). Equally important is awareness that, due to the pervasive and persistent nature of beliefs, unexamined teacher beliefs may thwart other efforts to improve mathematics education for the students they serve.

Conclusion

For the first time in our nation’s history, most children are learning the same mathematics content at the same grade-level. The development and widespread adoption of the CCSSM (NGA & CCSSO, 2010) in the United States reflects the desire that students in all states and territories have an equal opportunity to learn common and core mathematical knowledge. Common standards, however, do not ensure a common learning experience for students in all mathematics classrooms.

Teachers play a crucial role in determining how these standards are enacted.

This study confirms the finding that teacher beliefs influence practice; teachers who hold different mathematics beliefs assign different meanings to the language in mathematics standards and use different pedagogical approaches when teaching the standards.

Examining teacher beliefs, therefore, is a worthwhile focus of research because such beliefs are directly related to student learning. Mathematics education researchers have found that teachers' beliefs about their students' thinking are closely related to student achievement (Carpenter, Fennema, Peterson, & Carey, 1988). Systemic improvement in American mathematics education must include a better understanding of the complex relationship between teachers' mathematics beliefs and the ways they interpret and implement the CCSSM. By focusing on beliefs, perhaps "the single most important construct in educational research" (Fenstermacher, 1979 as cited in Pajares, 1992, p. 329), mathematics education researchers can indeed make valuable contributions toward the ultimate goal of improved mathematics teaching and learning for all.

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Appendix A: Demographic Questionnaire

ID # _____

Mathematics Beliefs Scales
Accompanying Questionnaire**TEACHING EXPERIENCE**Current Teaching Position(s)

Please circle all that apply.

classroom intervention support enrichment special education other

Current Grade Level(s)

Please circle all that apply.

PK K 1 2 3 4 5 6 7 8 9 10 11 12

Previous Teaching Position(s)

Please circle all that apply.

classroom intervention support enrichment special education other

Previous Grade Level(s)

Please circle all that apply.

PK K 1 2 3 4 5 6 7 8 9 10 11 12

Years teaching in the district _____

Years teaching in other districts _____

Total years teaching _____

ID # _____

LICENSURE

Please indicate the area(s) of licensure you currently hold.

EDUCATION

Please indicate degrees completed or in progress as well as major and minor courses of study.

<u>Degree(s)</u>	<u>Major(s)</u>	<u>Minor(s)</u>
<hr/>	<hr/>	<hr/>
<hr/>	<hr/>	<hr/>
<hr/>	<hr/>	<hr/>
<hr/>	<hr/>	<hr/>

MATHEMATICS PROFESSIONAL DEVELOPMENT & TRAININGPlease circle any professional development and/or training classes you have taken in **MATHEMATICS**.

workshops	curriculum training	standards training	conferences
university courses	online sessions	continuing education courses	

Appendix B: Survey

ID # _____

MATHEMATICS BELIEFS SCALES

(Fennema, Carpenter, & Loef, 1990)

A=Strongly Agree**B=Agree****C=Undecided****D=Disagree****E=Strongly Disagree**

- _____ 1. Children should solve word problems before they master computational procedures.
- _____ 2. Teachers should encourage children to find their own solutions to math problems even if they are inefficient.
- _____ 3. Children should understand computational procedures before children spend much time practicing computational procedures.
- _____ 4. Time should be spent solving simple word problems before children spend much time practicing computational procedures.
- _____ 5. Teachers should teach exact procedures for solving word problems.
- _____ 6. Children should understand the meaning of an operation (addition, subtraction, multiplication, or division) before they memorize number facts.
- _____ 7. The teacher should demonstrate how to solve simple word problems before children are allowed to solve word problems.
- _____ 8. The use of key words is an effective way for children to solve word problems.
- _____ 9. Mathematics should be presented to children in such a way that they can discover relationships for themselves.
- _____ 10. Even children who have not learned basic facts can have effective methods for solving problems.
- _____ 11. It is important for a child to be a good listener in order to learn how to do mathematics.
- _____ 12. Most young children can figure out a way to solve simple word problems.
- _____ 13. Children should have many informal experiences solving simple word problems before they are expected to memorize number facts.
- _____ 14. An effective teacher demonstrates the right way to do a word problem.
- _____ 15. Children should be told to solve problems the way the teacher has taught them.
- _____ 16. Most young children have to be shown how to solve simple word problems.

ID # _____

MATHEMATICS BELIEFS SCALES

(Fennema, Carpenter, & Loef, 1990)

A=Strongly Agree**B=Agree****C=Undecided D=Disagree****E=Strongly Disagree**

- ___ 17. Children's written answers to paper-and-pencil mathematical problems indicate their level of understanding.
- ___ 18. The best way to teach problem solving is to show children how to solve one kind of problem at a time.
- ___ 19. It is better to provide a variety of word problems for children to solve.
- ___ 20. Children learn math best by figuring out for themselves the ways to find answers to simple word problems.
- ___ 21. Children usually can figure out for themselves how to solve simple word problems.
- ___ 22. Recall of number facts should precede the development of an understanding of the related operation (addition, subtraction, multiplication, or division).
- ___ 23. Children will not understand an operation (addition, subtraction, multiplication, or division) until they have mastered some of the relevant number facts.
- ___ 24. Most children cannot figure math out for themselves and must be explicitly taught.
- ___ 25. Children should understand computational procedures before they master them.
- ___ 26. Children learn math best by attending to the teacher's explanations.
- ___ 27. It is important for a child to discover how to solve simple word problems for him/herself.
- ___ 28. Children should be allowed to invent new ways to solve simple word problems before the teacher demonstrates how to solve them.
- ___ 29. Time should be spent practicing computational procedures before children are expected to understand the procedures.
- ___ 30. The goals of instruction in mathematics are best achieved when students find their own methods for solving problems.
- ___ 31. Allowing children to discuss their thinking helps them to make sense of mathematics.
- ___ 32. Teachers should allow children who are having difficulty solving a word problem to continue to try to find a solution.

ID # _____

MATHEMATICS BELIEFS SCALES

(Fennema, Carpenter, & Loef, 1990)

A=Strongly Agree**B=Agree****C=Undecided D=Disagree****E=Strongly Disagree**

- ___ 33. Children can figure out ways to solve many math problems without formal instruction.
- ___ 34. Teachers should tell children who are having difficulty solving a word problem how to solve the problem.
- ___ 35. Frequent drills on the basic facts are essential in order for children to learn them.
- ___ 36. Most young children can figure out a way to solve many mathematics problems without any adult help.
- ___ 37. Teachers should allow children to figure out their own ways to solve simple word problems.
- ___ 38. It is better to teach children how to solve one kind of word problem at a time.
- ___ 39. Children should not solve simple word problems until they have mastered some number facts.
- ___ 40. Children's explanations of their solutions to problems are good indicators of their mathematics learning.
- ___ 41. Given appropriate materials, children can create meaningful procedures for computation.
- ___ 42. Time should be spent practicing computational procedures before children spend much time solving problems.
- ___ 43. Teachers should facilitate children's inventions of ways to solve simple word problems.
- ___ 44. It is important for a child to know how to follow directions to be a good problem solver.
- ___ 45. To be successful in mathematics, a child must be a good listener.
- ___ 46. Children need explicit instruction on how to solve word problems.
- ___ 47. Children should master computational procedures before they are expected to understand how those procedures work.
- ___ 48. Children learn mathematics best from teachers' demonstrations and explanations.

Appendix C: Interview Protocol

Common Core State Standards for Mathematics

(NGA & CCSSO, 2010)

Grade 2

- (2) Students use their understanding of addition to develop **fluency** with addition and subtraction within 100. They solve problems within 1000 by applying their understanding of models for addition and subtraction, and they develop, discuss, and use **efficient, accurate, and generalizable methods to compute sums and differences of whole numbers** in base-ten notation, using their understanding of place value and the properties of operations. They select and accurately apply methods that are appropriate for the context and the numbers involved to mentally calculate sums and differences for numbers with only tens or only hundreds. (Introduction, p. 18)

Number and Operations in Base Ten

2.NBT

Use place value understanding and properties of operations to add and subtract.

7. Add and subtract within 1000, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method. Understand that in adding or subtracting three digit numbers, one adds or subtracts hundreds and hundreds, tens and tens, ones and ones; and sometimes it is necessary to compose or decompose tens or hundreds. (Standards, p. 19)

Grade 2 Interview Questions

- 1) What does the word *fluency* mean to you? How does a student demonstrate *fluency*?
- 2) Placed together in the standard, what do the words *efficient*, *accurate*, and *generalizable methods to compute sums and differences of whole numbers* mean to you?
 - a) Describe what the word *efficient* means to you? What makes a method *efficient*?
 - b) What do the words *generalizable methods* mean to you? Why is the word *methods* plural?
- 3) What prior skills and understanding do your students need to add and subtract within 1000?
- 4) What strategies do your students use to add and subtract within 1000? How do your students learn these strategies?
- 5) Describe how students demonstrate their skills and understanding of adding and subtracting within 1000?
- 6) What is the relationship among skills (basic addition/subtraction facts and computation procedures), conceptual understanding (what addition and subtraction are), and problem solving?
 - a) Should children master addition and subtraction facts/computation procedures *before* they solve word problems involving these skills or can experiences solving such problems help children learn these skills?
 - b) Can children who have not mastered basic addition and subtraction facts/computational procedures solve word problems involving these skills? If so, how?
- 7) Describe your role in teaching students 2.NBT 7?

Common Core State Standards for Mathematics

(NGA & CCSSO, 2010)

Grade 3

- (1) Students develop an understanding of the meanings of multiplication and division of whole numbers through activities and problems involving equal-sized groups, arrays, and area models; multiplication is finding an unknown product, and division is finding an unknown factor in these situations. For equal-sized group situations, division can require finding the unknown number of groups or the unknown group size. Students use properties of operations to calculate products of whole numbers, **using increasingly sophisticated strategies based on these properties to solve multiplication and division problems** involving single-digit factors. By comparing a variety of solution strategies, students learn the relationship between multiplication and division. (Introduction, p. 21)

Operations and Algebraic Thinking

3.OA

Multiply and divide within 100.

7. **Fluently** multiply and divide within 100, using strategies such as the relationship between multiplication and division (e.g. knowing that $8 \times 5 = 40$, one knows $40 \div 5 = 8$) or properties of operations. By then end of Grade 3, know from memory all products of two one-digit numbers. (Standard, p. 23)

Grade 3 Interview Questions

- 1) What does the word *fluency* mean to you? How does a student demonstrate *fluency*?
- 2) Placed together in the standard, what do the words *increasingly sophisticated strategies based on these properties to solve multiplication and division problems* mean to you?
 - a) Describe what the word *sophisticated* means to you? What makes a method *sophisticated*?
 - b) What does the word *strategies* mean to you? Why is the word plural?
- 3) What prior skills and understanding do your students need to multiply and divide numbers within 100?
- 4) What strategies do your students use to multiply and divide numbers within 100? How do your students learn these strategies?
- 5) Describe how students demonstrate their skills and understanding of multiplying and dividing numbers within 100?
- 6) What is the relationship among skills (basic multiplication and division facts and computation procedures), conceptual understanding (what multiplication and division are), and problem solving?
 - a) Should children master multiplication and division facts/computation procedures *before* they solve word problems involving these skills or can experiences solving such problems help children learn these skills?
 - b) Can children who have not mastered basic multiplication and division facts/computational procedures solve word problems involving these skills? If so, how?
- 7) Describe your role in teaching students 3.OA 7?

Common Core State Standards for Mathematics

(NGA & CCSSO, 2010)

Grade 4

- (1) Students generalize their understanding of place value to 1,000,000, understanding the relative sizes of numbers in each place. They apply their understanding of models for multiplication (equal-sized groups, arrays, area models), place value, and properties of operations, in particular the distributive property, as they develop, discuss, and use efficient, accurate, and generalizable methods to compute products of multi-digit whole numbers. Depending on the numbers and the context, they select and accurately apply appropriate methods to estimate or mentally calculate products. They develop **fluency with efficient procedures for multiplying whole numbers**; understand and explain why the procedures work based on place value and properties of operations; and use them to solve problems. Students apply their understanding of models for division, place value, properties of operations, and the relationship of division to multiplication as they develop, discuss, and use efficient, accurate, and generalizable procedures to find quotients involving multi-digit dividends. They select and accurately apply appropriate methods to estimate and mentally calculate quotients, and interpret remainders based upon the context.

Number and Operations in Base Ten

4.NBT

Use place value understanding and properties of operations to perform multi-digit arithmetic.

5. Multiply a whole number of up to four digits by a one-digit whole number, and multiply two two-digit numbers, using strategies based on place value and the properties of operations. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.

Grade 4 Interview Questions

- 1) What does the word *fluency* mean to you? How does a student demonstrate *fluency*?
- 2) Placed together in the standard, what do the words *efficient procedures for multiplying whole numbers* mean to you?
 - a) Describe what the word *efficient* means to you? What makes a method *efficient*?
 - b) What does the word *procedures* mean to you? Why is the word plural?
- 3) What prior skills and understanding do your students need to multiply two two-digit numbers?
- 4) What strategies do your students use to multiply two two-digit numbers? How do your students learn these strategies?
- 5) Describe how students demonstrate their skills and understanding of multiplying two two-digit numbers?
- 6) What is the relationship among skills (basic multiplication facts and computation procedures), conceptual understanding (what multiplication is), and problem solving?
 - a) Should children master multiplication facts/computation procedures *before* they solve word problems involving these skills or can experiences solving such problems help children learn these skills?
 - b) Can children who have not mastered basic multiplication facts/computational procedures solve word problems involving these skills? If so, how?
- 7) Describe your role in teaching students 4.NBT 5?

Appendix D: Informed Consent

13 February, 2015

Dear Teacher,

This school district is participating in a study to gather information related to teachers' mathematical beliefs and the ways they interpret and implement the Common Core State Standards for Mathematics (CCSSM). The researcher, Erin Smith, is a public school teacher and doctoral candidate at Hamline University in St. Paul, MN. This project has been approved by Hamline University's School of Education Human Subjects Committee and is expected to be completed by August 31, 2015.

Your participation involves completing a demographic questionnaire and a survey called the Mathematics Beliefs Scales (Fennema, Carpenter, & Loef, 1990) during February or March, 2015 at your school. Based on survey results, six teachers will be invited to participate in individual interviews (lasting up to one hour) to discuss a CCSSM grade-specific standard.

Your participation in this study will contribute knowledge to the mathematics education field about the relationship between teacher beliefs and practices relating to the CCSSM. The district will be given a summary of research results. There are no costs for participating in the study other than time to complete the demographic questionnaire, survey, and (if selected) participate in a personal interview.

This research is public scholarship; the abstract and dissertation will be catalogued in Hamline University's Bush Library Digital Commons, a searchable electronic repository, and may be published or used in other ways. A number of steps will be taken to ensure privacy and confidentiality. Using identification numbers, all questionnaire and survey responses will be kept anonymous to the researcher during the first phase of the study. Interview participant identities will be kept strictly confidential. No names or other identifying information about individual teachers, schools, or the district will be used in the dissertation submitted to Hamline University or in any future material created about the study. Only general study findings will be reported.

Participation in this study is voluntary and the school district or any teacher may withdraw from the study at any time, for any reason, without penalty. The school district has consented to have this research conducted in its schools. If you have any questions or concerns about this study, now or in the future, please contact the researcher.

Sincerely,

Erin Smith
Hamline University Doctor of Education Student
651-592-0334 esmith01@hamline.edu

INFORMED CONSENT

Please keep this full page for your records.

I have been given the opportunity to read this consent form. I understand the information about this study. I understand that participation in the study involves the use of participant work in the form of questionnaire, survey, and interview responses. I agree to participate by completing a demographic questionnaire, a survey, and—if selected—a one-hour interview. Questions I had about the consent process and/or the study have been answered. My signature says that I am willing to participate in this study.

Teacher Participant
name printed

Teacher Participant
signature

Date

Investigator signature

Date

Participant Copy

INFORMED CONSENT

Please return this form to researcher Erin Smith.

I have been given the opportunity to read this consent form. I understand the information about this study. I understand that participation in the study involves the use of participant work in the form of questionnaire, survey, and interview responses. I agree to participate by completing a demographic questionnaire, a survey, and—if selected—a one-hour interview. Questions I had about the consent process and/or the study have been answered. My signature says that I am willing to participate in this study.

Teacher Participant
name printed

Teacher Participant
signature

Date

Investigator signature

Date

Researcher Copy

Appendix E: Additional Quantitative Data

Table 9

Raw Scores, Overall Mean Scores, and Standard Deviations

Participant ID	Grade Level	Raw Score	Grade Level Mean Raw Score	Overall Mean Score	Standard Deviation from Mean
220	K	141	160.86	2.94	-1
224	K	194	160.86	4.04	+2
230	1	149	171.85	3.10	-1
236	1	191	171.85	3.98	+1
254	2	141	164.82	2.94	-1
257	2	202	164.82	4.21	+1
124	3	131	164.00	2.73	-2
159	3	193	164.00	4.02	+1
154	4	138	165.00	2.88	-1
151	4	209	165.00	4.35	+2
172	5	156	181.43	3.25	-1
102	5	212	181.43	4.42	+1
105	6	149	172.80	3.10	-1
170	6	205	172.80	4.27	+2
142	M	152	173.67	3.17	-1
158	M	210	173.67	4.38	+2

Appendix F: Coding and Categorization of Qualitative Data

The traditional and reform mathematics education models, described in the literature review and summarized in Figure 2, provided a framework for placing teachers along a constructivist continuum in the quantitative research phase and shaped the codes and categories used in the qualitative data analysis. Traditional beliefs about mathematics, teaching, and learning are less aligned with constructivist learning theory while reform (non-traditional) beliefs about mathematics, teaching, and learning are more aligned with constructivist learning theory.

Table 10

Implementation Practices Related to Beliefs about Mathematics

Category: Relationship Among Procedural Skills, Conceptual Understanding and Word Problem Solving		
Code	Description	Belief LC = lower constructivist HC = higher constructivist
BF	basic facts (first and foremost)	LC
SA	standard algorithm encouraged	LC
PS	procedural steps emphasized	LC
SPB	skills/procedures before problem solving	LC
BFO	basic facts as one part of mathematics	HC
MA	multiple algorithms encouraged	HC
US	understanding of algorithm emphasized	HC
SPI	skills/procedures integrated with word problems	HC

Table 11

Implementation Practices Related to Beliefs about Learning Mathematics

Category: How Students Learn Mathematics		
Code	Description	Belief LC = lower constructivist HC = higher constructivist
SL	students listen to teacher voice	LC
SP	students practice using teacher model	LC
SFS	students following steps of procedure	LC
SMMT	students using manipulatives/models/tools	LC/HC
STP	students teaching peers	HC
SD	student discovery	HC
CA	classroom activities	HC
SV	student voices involved in learning	HC

Table 12

Implementation Practices Related to Beliefs about Teaching Mathematics

Category: How Mathematics Should Be Taught		
Code	Description	Belief LC = lower constructivist HC = higher constructivist
TD	teacher demonstration	LC
TM	teacher modeling	LC
TDI	teacher direct instruction	LC
TE	teacher explanation	LC
TA	teacher authority	LC
TPO	teacher providing opportunities to learn	HC
TAF	teacher as facilitator	HC
PT	peer teaching	HC
IBT	inquiry-based teaching	HC
SMT	students' math. thinking guides instruction	HC
SRE	students give reasoning and explanations	HC
TCU	teaching for conceptual understanding	HC
BPK	building on prior knowledge	HC